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Abstract

We propose an approach for incorporating data on social interactions (e.g., number of recommendations received by consumers, number of recommendations given by adopters) into the calibration of diffusion models. In order to do so, we extend major extant diffusion models to capture explicitly the generation of social interactions and their impact on adoption. In particular, we extend the discrete-time versions of the Mixed Influence Model (Bass model), the Asymmetric Influence Model, and the Karmeshu-Goswami Model. The extended models may be calibrated using a combination of social interactions data and penetration data. Two field studies, collected in collaboration with a word of mouth marketing company, two Consumer Packaged Goods companies and two marketing research companies, suggest that the incorporation of social interactions data results in improved diffusion forecasts.

Keywords: diffusion models, forecasting, innovations, measurement, social networks.

1. Introduction

In the past few years, data on social interactions between consumers, including word-of-mouth (WOM) recommendations and other product-related interactions, have become increasingly available. For example, the Keller Fay group's TalkTrak® system (www.kellerfay.com) interviews a large, representative sample of the US population on a daily basis, asking participants to record all of their product-related conversations (both offline and online) for 24 hours. Word of mouth (WOM) marketing companies like BzzAgent™ (www.bzzagent.com) have assembled large panels of consumers who spread word of mouth about new products and report their interactions with other consumers to the company (Godes and Mayzlin 2009). Using a somewhat related business model, companies like shespeaks℠ (www.shespeaks.com), Tremor (www.tremor.com) and Vocalpoint (site.vocalpoint.com) have built communities of consumers who share similar interests. These companies offer their members early access to new products, allow them to share their opinions on these products through online discussion forums and various surveys, and routinely measure the recommendations made by their members to other consumers. Finally, market research firms like Nielsen Online (www.nielsen-online.com) mine millions of online communities, discussion boards, blogs and social networks to quantify online conversations about brands and products. Of course, these companies are not the only source of social interactions data, and such data may also be collected directly using traditional methods.

While social interactions and social influence have been studied extensively in marketing, applications of social interactions *data* such as the ones described above are more limited. Recent research has primarily focused on analyzing and quantifying the impact of social interactions on sales and diffusion. For example, Godes and Mayzlin (2009) study how WOM created in a viral marketing campaign influences sales, and how characteristics of the transmitter and of the recipient of WOM influence the effectiveness of these social interactions. Chevalier and Mayzlin (2006) study the impact of online book reviews on sales. Godes and Mayzlin (2004) study how the quantity and the dispersion of online discussions about new TV shows are linked to the shows' ratings. Trusov, Bucklin and Pauwels (2008) study how electronic invitations sent out by the members of an online social networking site impact the number of new users, relative to traditional marketing efforts.

The present paper offers a new application of social interactions data. In particular, our research looks at how to use social interactions data to improve the diffusion forecasts made by extant diffusion models. Because social influence is at the heart of diffusion models, it seems natural to assume that social interactions data may be useful in calibrating diffusion models and give rise to improved diffusion forecasts.

Many diffusion models used in marketing may be traced back to the Bass model (Bass 1969), also referred to as the Mixed Influence Model (Mahajan and Peterson, 1985), and its antecedents (e.g., Mansfield 1961). This model has been extended for example to account for asymmetric influence between different segments of potential adopters (Lehmann and Esteban-Bravo 2006; Muller and Yogev 2006; Van den Bulte and Joshi 2007), and heterogeneity across potential adopters (Karmeshu and Goswami 2001).

One of the most managerially relevant applications of diffusion models is forecasting future diffusion based on past data. However, extant diffusion models may not be calibrated reliably at early stages of the diffusion process based on aggregate penetration data only (see for example Hauser, Tellis, and Griffin, 2006; Mahajan, Muller, and Bass, 1990; Van den Bulte and Lilien, 1997). The common solution to this problem is to complement aggregate penetration data with other data. For example, the diffusion rate of past analogous innovations has been shown to be a useful source of complementary information (Bass et al. 2001; Hahn et al. 1994; Lenk and Rao 1990; Roberts, Nelson and Morrisson 2005; Sultan, Farley, and Lehmann 1990; Sood, James and Tellis 2009; Talukdar, Sudhir and Ainslie 2002). Data on the timing of adoption of consumers in a sample may also be used to inform parameter estimates (Schmittlein and Mahajan 1982; Sinha and Chandrashekaran 1992). In both cases, additional *penetration data* is used to complement aggregate penetration data. The framework proposed in this paper is not incompatible with the use of such ancillary data: it allows using data on social interactions, *in addition* to these other sources of data.

One challenge with incorporating social interactions data into diffusion forecasts is that it is not obvious *how* social interactions data may be incorporated in the calibration of extant diffusion models. Consider for example the hazard rate of the Mixed Influence Model, $h(t)=p+q.F(t)$, where p and q are the coefficients of external influence and the coefficient of internal influence respectively, and $F(t)$ is the cumulative penetration at time t. Suppose that data were available on the number of recommendations received by a set of consumers, as well as data on which of these consumers have adopted the innovation and how many recommendations these adopters gave in turn to other consumers. A likelihood function for these data may *not* be derived readily from the Mixed Influence Model. This is because the Mixed Influence Model, like most extant diffusion models, does not explicitly capture the probability of adopting conditional on a number of recommendations nor the generation of recommendations.

We address this challenge by extending major extant diffusion models so that they may be calibrated with a combination of penetration data and social interactions data. Precisely, we nest the discrete-time versions of the Mixed Influence Model (Bass model), the Asymmetric

Influence Model, and the Karmeshu-Goswami Model. The extended models capture explicitly the generation of social interactions as well as their impact on adoption. Using two field data sets collected in collaboration with a word of mouth marketing company, two Consumer Packaged Goods companies and a marketing research company, we illustrate how social interactions data may be combined with penetration data to calibrate the extended models, resulting in improved diffusion forecasts.

Previous attempts to model social interactions directly in diffusion models include Van den Bulte and Lilien (2001) who model the diffusion of the drug Tetracycline across a community of 121 physicians by capturing the structure of the physicians' social network, and modeling the effect on physician i of the adoption of another physician j to which i is connected. However this type of approach (see also for example Iyengar, Van den Bulte and Valente 2009; and Strang 1991) requires mapping the complete social network of the entire potential market (or at least a large proportion thereof). Such data are difficult to obtain for most consumer products. In contrast, our approach allows calibrating diffusion models using social interactions data from a sample of consumers without having to map the social network. Other researchers, mentioned above, have quantified the impact of online social interactions on the sales of new products. The approach used by these researchers differs from ours in that it is primarily descriptive, and therefore relies on models that do not produce out-of-sample diffusion forecasts.

This paper is organized as follows. In Section 2, we describe the extension of three major extant diffusion models (Mixed Influence Model, Asymmetric Influence Model, Karmeshu-Goswami Model). In Section 3, we illustrate how data on social interactions may be combined with penetration data to calibrate the extended models. We conclude in Section 4 and offer directions for future research.

2. Model Development

Our approach is to extend existing diffusion models that have been studied and validated by many researchers over a long period of time, rather than attempt to develop new, fundamentally different diffusion models. Our intent is not to replace extant models but to demonstrate how social interactions data may be used to supplement these long proven models. This ‘extension’ approach drives our modeling assumptions. In particular, we make assumptions that allow nesting extant diffusion models while deviating as little as possible from them, and we minimize the introduction of new assumptions not explicitly made by these models. When appropriate, we highlight how some of the assumptions may be relaxed or extended in future research.

For simplicity, in the remainder of the paper we focus on *recommendations* as the primary source of social interactions. When appropriate, we note how other forms of social interactions (e.g., observing other consumers using the innovation) may be captured as well, by modifying the definition of the parameters of the models.

For ease of exposition we start with the extension of the most popular diffusion model in marketing, the Mixed Influence Model (Bass 1969). We next turn to the more recent Asymmetric Influence Model (Van den Bulte and Joshi 2007) and Karmeshu-Goswami Model (Karmeshu and Goswami 2001). We summarize the parameters of the extended models in Table 1.

[INSERT TABLE 1 ABOUT HERE]

a. Extending the Discretized Mixed-Influence Model (Bass Model)

We first describe the modeling of the probability of adoption conditional on the number of recommendations received, and of the generation of recommendations. Next, we provide closed-form expressions for the resulting aggregate diffusion process, and show formally that the discretized Mixed Influence Model is nested.

Adoption conditional on number of recommendations received

The probability of adopting in period t conditional on receiving a number, n_t, of recommendations in period t and on not having adopted as of period t, may be modeled with a discrete-time conditional hazard rate $h(n_t)$. We will show later how the following specification allows nesting the Mixed Influence Model:

$$h_t(n_t) = 1 - (1-p).(1-q)^{n_t} \quad (1)$$

The parameters p and q capture similar forces as the parameters of the Mixed Influence Model, with p capturing external effects and q capturing internal effects. However, the parameter q is defined here as the probability that a potential adopter would adopt based on one recommendation, in the absence of external effects. The conditional hazard rate is equal to one minus the probability of "resisting" the innovation, which is equal to the probability of "resisting" the external forces and "resisting" the influence of n_t recommendations.[1]

Generation of recommendations

We now model the generation of recommendations, in a way that allows nesting the Mixed Influence Model. We model the number of recommendations received by a potential

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[1] The conditional hazard rate in Equation (1) is comparable to the hazard rates assumed by agent-based models (Garber et al. 2004; Goldenberg et al. 2002).

adopter in period t, n_t, using a binomial distribution. The number of trials equals the number of social ties this individual has and the success probability equals the probability that each of these ties would recommend the product to this person in period t. This latter probability is expressed as the probability that a given tie would recommend the product to this potential adopter conditional on the tie having adopted, multiplied by the probability that the tie has adopted.

Formallly:

$$n_t \sim Binomial(ties, a.F_{t-1}) \quad (2)$$

where the parameter a captures the probability that a consumer recommends the innovation to each of his or her ties in each period conditional on having adopted the innovation, and F_{t-1} is the cumulative penetration in period t-1, which is equal to the probability that a randomly selected consumer has adopted the innovation by period t- 1. The parameter *ties* captures the number of social ties between consumers.

We note that this simple specification may be extended or modified in at least three ways. First, heterogeneity in the number of social ties between consumers, *ties*, could be introduced, for example by assuming that it follows a discrete probability distribution. Second, the above specification assumes that the probability that an adopter will recommend the product to each of his or her ties is constant over time. Non-uniform influence (Easingwood, Mahajan and Muller 1983) may be captured by making the parameter a a function of the period at which the adoption occurred. Third, in some cases information may be available on the structure of the social network between consumers, and the above specification could be extended to reflect such information. While the first two extensions are straightforward, the third is more complex as it would make the binomial draws non-independent and may require the use of simulations. As mentioned above, in this paper we minimize the introduction of new assumptions not explicitly made by extant models. Therefore we leave such extensions to future research.

Finally, note that other forms of social interactions, different from recommendations, may be captured as well by modifying the definition of the parameter *a*. For example, if social influence works through potential adopters observing other consumers using the innovation, the parameter *a* may be defined as the probability that an adopter will be using the innovation while interacting with each of his or her ties.

Aggregate diffusion process

We now show how the individual-level mechanisms captured in equations (1) and (2) may be aggregated to obtain closed-form expressions of the aggregate diffusion process. The marginal penetration in period t, f_t, is equal to the proportion of non-adopters before period t multiplied by the expected value of the hazard rate in period t, where the expected value is taken over n_t. Formally:

$$f_t = (1-F_{t-1}).\underset{n_t}{E}[h(n_t)] \tag{3}$$

$$= (1-F_{t-1}).\sum_{n_t=0}^{ties}(1-(1-p).(1-q)^{n_t}).P(n_t)$$

$$= (1-F_{t-1}).\sum_{n_t=0}^{ties}(1-(1-p).(1-q)^{n_t}).\binom{ties}{n_t}(a.F_{t-1})^{n_t}.(1-a.F_{t-1})^{(ties-n_t)}$$

The above equation provides a closed-form expression of the marginal penetration in period t given the cumulative penetration in the previous period. Marginal penetration in any period unconditional on past penetration is obtained recursively, *without* using any simulation or numerical approximation.

Relation to Mixed Influence Model

Finally, we show how the discretized Mixed Influence Model may be obtained as a special case, in which the number of social ties is assumed to be 1. Under the assumption that *ties*=1, the number of recommendations received by a potential adopter in period t, n_t, is 1 with probability $a.F_{t-1}$ and 0 with probability $(1-a.F_{t-1})$. The expected value of the hazard rate over n_t becomes equal to the hazard rate of the discretized Mixed Influence Model, with $p^{Bass} \equiv p$ and $q^{Bass} \equiv q.(1-p).a$:

$$\begin{aligned} h_t &= \underset{n_t}{E}[h(n_t)] = \sum_{n_t=0}^{ties} (1-(1-p).(1-q)^{n_t}).P(n_t) = p.P(n_t=0) + (1-(1-p).(1-q)).P(n_t=1) \\ &= p.(1-a.F_{t-1}) + (p+q.(1-p)).a.F_{t-1} = p + q.(1-p).a.F_{t-1} \end{aligned} \tag{4}$$

Note that this special case is presented here only in order to establish that the model described in equations (1) to (3), which we will refer to as the extended Mixed Influence Model, nests the Mixed Influence Model. We will not set the parameter *ties* to 1 in our field applications.

As a step towards incorporating social interactions data, we have extended the discretized Mixed Influence Model to capture explicitly the generation of social interactions and their impact on adoption. We now use a similar approach to extend the Asymmetric Influence Model and the Karmeshu-Goswami Model.

b. Extending the Discretized Asymmetric Influence Model

The Mixed Influence Model is probably the best-known diffusion model in marketing, and it has been used in a large number of applications. However, many theoretical developments have occurred in the forty years since it was introduced. One of the latest models proposed in the literature, which is viewed by many as the state of the art in marketing diffusion models, is the

Asymmetric Influence Model of Van den Bulte and Joshi (2007). This model assumes the existence of two segments with asymmetric influence on one another (see also Lehmann and Esteban-Bravo 2006, and Muller and Yogev 2006).[2] These two segments, labeled as "innovators" and "imitators," are such that innovators are only influenced by other innovators, while imitators are influenced both by innovators and by imitators. Following Van den Bulte and Joshi (2007), we refer to the innovators segment as segment 1 and to the imitators segment as segment 2. Like in the original model, we assume that the proportion of innovators in the potential market is given by θ. Following the previous format, we first describe the modeling of adoption conditional on the number of recommendations, followed by the modeling of the generation of recommendations. Next, we provide closed form expressions for the aggregate diffusion process and verify that the Asymmetric Influence Model is indeed nested.

Adoption conditional on number of recommendations received

Adoption conditional on the number of recommendations received in the innovators segment is modeled similarly to Equation 1:

$$h_t^1(n_t^{1-1}) = 1-(1-p_1).(1-q_1)^{n_t^{1-1}} \quad (5)$$

where $n_t^{1\text{-}1}$ refers to recommendations from innovators to innovators.

The probability of adoption conditional on the number of recommendations received in the imitators segment is modeled similarly, with the exception that imitators receive recommendations from both innovators and other imitators:

$$h_t^2(n_t^{1-2}, n_t^{2-2}) = 1-(1-p_2).(1-q_2)^{n_t^{1-2}+n_t^{2-2}} \quad (6)$$

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[2] We refer the readers to Van den Bulte and Joshi (2007) for a review of the literature supporting the existence of two such segments.

where $n_t^{1\text{-}2}$ refers to recommendations from innovators to imitators and $n_t^{2\text{-}2}$ refers to recommendations from imitators to imitators. The parameter p_1 and p_2 capture external effects in segments 1 and 2 respectively. The parameter q_1 (respectively q_2) is the probability that an innovator (respectively imitator) would adopt based on one recommendation, in the absence of external effects.

Generation of recommendations

The generation of recommendations is modeled as follows:

$$n_t^{1-1} \sim Binomial(ties^{1-1}, a^{1-1}.F_{t-1}^{1}) \quad (7)$$

$$n_t^{1-2} \sim Binomial(ties^{1-2}, a^{1-2}.F_{t-1}^{1}) \quad (8)$$

$$n_t^{2-2} \sim Binomial(ties^{2-2}, a^{2-2}.F_{t-1}^{2}) \quad (9)$$

where the parameter $ties^{i\text{-}j}$ captures the number of social ties between consumers in segment i and j, and the parameter $a^{i\text{-}j}$ captures the probability that an adopter from segment i would recommend the product to each of his or her ties from segment j in each subsequent period.

Aggregate diffusion process

Closed-form expressions of the aggregate diffusion process are obtained as follows:

$$\begin{aligned} f_t^1 &= (1-F_{t-1}^1).\underset{n_t^{1-1}}{E}[h^1(n_t^{1-1})] \\ &= (1-F_{t-1}^1).\sum_{n_t^{1-1}=0}^{ties^{1-1}}(1-(1-p_1).(1-q_1)^{n_t^{1-1}}).\binom{ties^{1-1}}{n_t^{1-1}}.(a^{1-1}.F_{t-1}^1)^{n_t^{1-1}}.(1-a^{1-1}.F_{t-1}^1)^{(ties^{1-1}-n_t^{1-1})} \end{aligned} \quad (10)$$

$$f_t^2 = (1-F_{t-1}^2).\underset{n_t^{1-2},n_t^{2-2}}{E}[h^2(n_t^{1-2},n_t^{2-2})]$$

$$= (1-F_{t-1}^2).\sum_{n_t^{1-2}=0}^{ties^{1-2}}\sum_{n_t^{2-2}=0}^{ties^{2-2}}(1-(1-p_2).(1-q_2)^{n_t^{1-2}+n_t^{2-2}}).$$

$$\binom{ties^{1-2}}{n_t^{1-2}}.(a^{1-2}.F_{t-1}^1)^{n_t^{1-2}}.(1-a^{1-2}.F_{t-1}^1)^{(ties^{1-2}-n_t^{1-2})}.\binom{ties^{2-2}}{n_t^{2-2}}.(a^{2-2}.F_{t-1}^2)^{n_t^{2-2}}.(1-a^{2-2}.F_{t-1}^2)^{(ties^{2-2}-n_t^{2-2})}$$

Relation to Asymmetric Influence Model

Consider the special case in which $ties^{1\text{-}1}$=1 and $\{ties^{1\text{-}2},ties^{2\text{-}2}\}=\{1,0\}$ with probability w, and $\{0,1\}$ with probability (1-w). This corresponds to a situation in which each innovator has one social tie with another innovator, and each imitator has one social tie, which is with an innovator with probability w or with an imitator with probability (1-w). In that case, the expected hazard rates are as follows:

$$h_t^1 = \underset{n_t^{1-1}}{E}[h_t^1(n_t^{1-1})] = p_1.(1-a^{1-1}.F_{t-1}^1)+(1-(1-p_1).(1-q_1)).(a^{1-1}.F_{t-1}^1)$$
$$= p_1 + q_1.(1-p_1).a^{1-1}.F_{t-1}^1 \qquad (11)$$

$$h_t^2 = \underset{n_t^{1-1},n_t^{1-2}}{E}[h^2(n_t^{1-2},n_t^{2-2})] = w.[p_2+q_2.(1-p_2).a^{1-2}.F_{t-1}^1]+(1-w).[p_2+q_2.(1-p_2).a^{2-2}.F_{t-1}^2]$$
$$= p_2 + q_2.(1-p_2).[w.a^{1-2}.F_{t-1}^1+(1-w).a^{2-2}.F_{t-1}^2] \qquad (12)$$

These hazard rates are identical to those of the discretized Asymmetric Influence Model (AIM), with $p^{1,AIM}\equiv p_1$, $q^{1,AIM}\equiv q_1.(1\text{-}p_1).a^{1\text{-}1}$, $p^{2,AIM}=p_2$, $q^{2,AIM}\equiv q_2.(1-p_2).[w.a^{1-2}+(1-w).a^{2-2}]$,

$w^{AIM} = \dfrac{w.a^{1-2}}{w.a^{1-2}+(1-w).a^{2-2}}$. (The readers are referred to Van den Bulte and Joshi 2007, page 402, for the notations of the Asymmetric Influence Model.)

c. Extending the Discretized Karmeshu-Goswami Model

Karmeshu and Goswami (2001) proposed an extension of the Mixed Influence Model that assumes heterogeneity in *p* and *q*, such that these two parameters are uncorrelated and that diffusion within a segment of the population defined by a specific pair of values (*p*,*q*) is independent from the other segments.[3] In other words, diffusion in each segment is captured by the Mixed Influence Model with the values of *p* and *q* corresponding to that segment. The model from Section 2.a may be extended in a similar fashion. The resulting diffusion process is as follows, where *g* refers to the probability distribution on *p* and *q*, and the diffusion in each segment is given by Equation (3):

$$
\begin{aligned}
f_t &= g(p_{low}, q_{low}).f_t^{low-low} + g(p_{high}, q_{low}).f_t^{high-low} \\
&+ g(p_{low}, q_{high}).f_t^{low-high} + g(p_{high}, q_{high}).f_t^{high-high}
\end{aligned}
\qquad (13)
$$

As in Section 2.a, the discretized Karmeshu and Goswami model is a special case in which the number of social ties, *ties*, is set to 1.

In this section we have extended the Mixed Influence Model, the Asymmetric Influence Model, and the Karmeshu-Goswami Model to capture explicitly the probability of adopting conditional on recommendations and the generation of recommendations. In the next section we describe two field studies, which illustrate how social interactions data may be combined with penetration data to calibrate these extended models.

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[3] Karmeshu and Goswami (2001) also proposed a different version of their model in which *p* and *q* are correlated. This version may also be nested using the proposed approach.

3. Field Studies

Our primary goal in this section is to demonstrate the feasibility, and evaluate the benefits of combining social interactions data with penetration data, and not to compare the Mixed Influence Model, the Asymmetric Influence Model and the Karmeshu-Goswami Model to one another. In particular, we expect that for each of these three diffusion models, combining social interactions data with penetration data using the extended version of the model will give rise to improved diffusion forecasts over those obtained from penetration data only.

a. Study 1

Set up and data

Our data were collected in collaboration with the Israeli word of mouth marketing company WOM (www.wom.co.il) and Hogla-Kimberly, a CPG company. WOM has assembled an online panel of women, who receive early access to new products and are given opportunities to share their feedback on these products with companies. WOM was involved in the launch a new feminine hygiene product offered by Hogla-Kimberly. This product offered a significant new benefit and represented an innovation in that category. For confidentiality reasons, we will refer to this new product as PROD1.

We define one time period as 12 weeks. By definition, the product was launched before t=1. During t=5, free samples of the product were sent to WOM members. At the end of this period, WOM asked these members to forward a questionnaire to the consumers to whom they had recommended the product. (The questionnaire was not designed specifically for the purpose of this project.) Our social interactions data focus on these recipients, i.e., consumers who received at least one recommendation for the product from a WOM member, but who were not

WOM members themselves. One of the items in the questionnaire asked the respondent whether she had heard about the product from another consumer. We limit our analysis to the subset of respondents who had not heard about the product from any other consumer besides the WOM member (N=321). Therefore, our sample consists of consumers who received exactly one recommendation for the product during t=1,…5, which came from a WOM member in period t=5.[4] Our social interactions data come from the following two items in the questionnaire:

- "Did you buy the product?" (Yes / No). Let y_i=1 if respondent i answered "yes" to that question, and 0 otherwise. A total of 162 respondents answered "yes" to that question.
- If y_i=1: "To how many people have you recommended the product so far?" (numerical answer). Let d_i be the number entered by respondent i, which in our data varied between 0 to 12.

In addition to these data, we received aggregate penetration data from Hogla-Kimberly (collected by Nielsen Israel). These data track the number of households in the market who purchased the product for the first time in t=1,…7. We use t=1,..5 as calibration periods and t=6 and 7 as validation periods. Let S_t be the number of households in the market who purchased the product for the first time in period t. Finally, we were provided with the number of households in the market with at least one woman, which we denote by M and refer to as the target market size (which is different from m, the number of eventual adopters – see below).

Calibration

We calibrate the original Mixed Influence Model, Asymmetric Influence Model and Karmeshu-Goswami Model using the aggregate penetration data. We calibrate the corresponding

[4] We do not use data from the consumers who had received other recommendations, as the questionnaire did not collect the number of other recommendations received or the times at which they were received.

extended models using both the aggregate penetration data and the social interactions data. We use a similar procedure (Bayesian MCMC) for all models.

<u>Extended and Original Mixed Influence Model</u>

We first describe the calibration of the extended Mixed Influence Model (referred to with the superscript *EXT-MIM*). In order to calibrate the model, we write likelihood functions for each piece of social interactions data, as well for the aggregate penetration data.

Let us first consider y_i, which indicates whether consumer i purchased the product for the first time between t=1 and t=5. The respondents in our sample received one recommendation during these time periods, which happened in t=5. Based on Equation (1), the hazard rate for these consumers is p in periods t=1 to t=4, and it is 1-(1-p)(1-q) in period t=5. Therefore, the cumulative hazard rate is:

$$P^{EXT-MIM}(y_i = 1) = 1 - (1-p)^5(1-q) \tag{14}$$

We now turn to the other piece of social interactions data, the number of recommendations given, d_i. Consider the subset of respondents for whom y_i=1, which represents 162 respondents. Recall that the parameter a in the model captures the probability that a consumer who has adopted the innovation recommends it to each of her social ties in each period. Suppose that consumer i adopted the product in period t_i. In each of the subsequent periods (t_i+1,..5), the number of recommendations given by this consumer is given by a binomial distribution: *Binomial*(*ties*,a). The number of recommendations given by this consumer over periods t_i+1 to 5 is therefore given by a binomial distribution: *Binomial*(*ties*.(5-t_i), a). Because we do not have information on when consumer i adopted the product, we need to integrate over t_i=1,..5. In order to do this, we need to calculate the probability that a consumer adopted in a specific time period, t_i, (where t_i=1…5) given that she adopted at some point over the first 5 periods and that she received one recommendation in period t=5. This probability is given by:

$\frac{(1-p)^{t_i-1}.(1-(1-p).(1-q)^{1(t_i=5)})}{1-(1-p)^5.(1-q)}$, where $(1-p)^{t_i-1}$ is the probability of not adopting over the first t_i-1 periods, and $(1-(1-p).(1-q)^{1(t_i=5)})$, where 1($t_i$=5) is 1 if t_i=5 and 0 otherwise, is the probability of adopting in period t_i, conditional on not having adopted in the first t_i-1 periods and on receiving one recommendation in t=5. Therefore, the likelihood contribution for d_i is given by:

$$P^{EXT-MIM}(d_i \mid y_i = 1) = \sum_{t_i=1}^{5} \binom{ties.(5-t_i)}{d_i} .(1-a)^{(ties.(5-t_i)-d_i)} .a^{d_i} . \frac{(1-p)^{t_i-1}.(1-(1-p).(1-q)^{1(t_i=5)})}{1-(1-p)^5.(1-q)} \qquad (15)$$

Finally, we specify a likelihood function for the aggregate penetration data. We simply assume that the aggregate penetration in period t, S_t, is equal to the penetration predicted by the model, plus a normal i.i.d. noise:

$$S_t = m.f_t^{EXT-MIM} + \varepsilon_t, \qquad \varepsilon_t \sim N(0,\sigma^2) \qquad (16)$$

where S_t is the measured number of adopters in period t, m is the number of eventual adopters (which is a subset of the target market - the estimation of m is described below), and $f_t^{EXT-MIM}$ is obtained by applying Equation (3) recursively.

Combining the social interactions data and the aggregate penetration data gives rise to the following model:

$$P^{EXT-MIM}(y_i = 1) = 1-(1-p)^5(1-q)$$

$$P^{EXT-MIM}(d_i \mid y_i) = \sum_{t_i=1}^{5} \binom{ties.(5-t_i)}{d_i} .(1-a)^{(ties.(5-t_i)-d_i)} .a^{d_i} . \frac{(1-p)^{t_i-1}.(1-(1-p).(1-q)^{1(t_i=5)})}{1-(1-p)^5.(1-q)}$$

$$S_t = m.f_t^{EXT-MIM} + \varepsilon_t, \qquad \varepsilon_t \sim N(0,\sigma^2)$$

We estimated this model using Bayesian MCMC (Rossi and Allenby, 2003), with the following uninformative priors: $\sigma^2 \sim IG(\frac{r_0}{2},\frac{s_0}{2})$ with $r_0 = s_0 = 1$, *p*, *q* and *a* uniform on [0,1], *ties* uniform on [1,50], and *m* uniform on [0,*M*]. The Metropolis-Hastings algorithm was used for all the parameters, except for *σ* which was drawn directly from its (inverse-gamma distributed) conditional posterior distribution. We used 100,000 MCMC iterations, using the first 50,000 as burn-in and saving 1 in every 10 draws. Convergence was assessed informally through time-series plots of the parameters. Similar priors and numbers of draws were used for all models.

The original Mixed Influence Model was estimated using a similar procedure, with the important exception that only aggregate penetration data were used in that case. In particular, the model becomes:

$$S_t = m.f_t^{MIM} + \varepsilon_t \qquad \varepsilon_t \sim N(0,\sigma^2)$$

where $f_t^{MIM} = \frac{1-\exp(-(p+q).t)}{1+\frac{q}{p}.\exp(-(p+q).t)} - \frac{1-\exp(-(p+q).(t-1))}{1+\frac{q}{p}.\exp(-(p+q).(t-1))}$ is the penetration given by the Mixed Influence Model. This model was also estimated using Bayesian MCMC, with the same uninformative priors on *σ*, *p*, *q*, and *m* as above.[5]

Note that although y_i captures the adoption decision of the consumers in our sample, these data may not be used to calibrate the original Mixed Influence Model in this study. Indeed, all consumers in our sample received exactly one recommendation, and the specification of the

[5] While the ratio of the number of observations to the number of parameters is large when social interactions data are combined with penetration data, this ratio is close to 1 when aggregate penetration data only are used. However the key relevant quantities (e.g., cumulative penetration) converged well in all cases.

hazard rate of the Mixed Influence Model does not allow conditioning on this selection criterion. In the next study, however, our sample will not be constructed in such a manner, and y_i will be used to calibrate the original Mixed Influence Model, Asymmetric Influence Model, and Karmeshu-Goswami Model.

Asymmetric Influence Model and Karmeshu-Goswami Model

The calibration of the extended and original Asymmetric Influence Model and Karmeshu-Goswami Model followed a similar approach, with each original model calibrated based on the aggregate penetration data and each extended model calibrated based on the social interactions data and the aggregate penetration data. One difference is that these models rely on the existence of different types of consumers. Because our social interactions data come from a sample of consumers selected according to some criteria (e.g., they had all received exactly one recommendation during t=5), we compute posterior probabilities of belonging to the various types conditional on the selection criteria.

Following Van den Bulte and Joshi (2007), we limit ourselves to two special cases of the Asymmetric Influence Model: a) one in which the innovators and imitators segments are independent (w=0 in the original Asymmetric Influence Model; $ties^{1\text{-}2} = a^{1\text{-}2} = 0$ in the extended model); and b) one in which innovators are not subject to social influence (q_1=0 in the original Asymmetric Influence Model; $q_1 = a^{1\text{-}1} = ties^{1\text{-}1} = 0$ in the extended model[6]). This allows avoiding the problematic computation of Gaussian hypergeometric functions in the original Asymmetric Influence Model (Van Den Bulte and Joshi 2007, page 412). Following Van Den Bulte and Joshi (2007), we retain the special case that provides the best fit with the calibration data.

[6] Special case b) of the original Asymmetric Influence Model may also be interpreted as $q_1 = 0$, $a^{1\text{-}1} \neq 0$, $ties^{1\text{-}1} \neq 0$. We use $q_1 = a^{1\text{-}1} = ties^{1\text{-}1} = 0$ for simplicity.

We describe the likelihood contribution of each piece of social interactions data in Appendix 1 (extended Asymmetric Influence Model) and Appendix 2 (extended Karmeshu-Goswami Model).

Out-of-sample predictive performance

For each model, we simulate one adoption curve for each of the 5,000 saved MCMC draws, using the corresponding parameter values. From each MCMC draw and its corresponding adoption curve, we extract the total number of adopters in the holdout validation periods (t=6,7), expressed in percent of the target market M. The posterior distribution of this quantity is approximated by its distribution across the MCMC draws. We quantify out-of-sample predictive performance by computing for each model the mean squared error between the true and predicted values, where the mean is taken over the posterior draws. A lower mean squared error is achieved when the mean of the posterior is closer to the true value, and/or the variance of the posterior is smaller.

The results are reported in Table 2. As predicted, the out-of-sample predictive ability corresponding to the extended version of each model (which combines social interactions data with penetration data) is lower than that corresponding to the original model (which relies on aggregate penetration data only), reflecting improved diffusion forecasts. Figure 2 plots the posterior distributions of the total number of adopters in the validation periods for each model, and suggests that the lower mean squared error is at least partly driven by the fact that the posterior distribution has a lower variance. The true value is always contained in the 95% credible interval for each extended and original model. However the *width* of the 95% credible interval obtained from the extended Mixed Influence Model (respectively, the Asymmetric Influence Model and the Karmeshu-Goswami Model) calibrated with social interactions and penetration data is 28% (respectively 47% and 49%) of the width of the 95% credible interval obtained from the original model calibrated with penetration data only.

Although our focus is on diffusion forecasts, for completeness we also report the mean squared error between the true and predicted cumulative number of adopters in the *calibration* periods (t=1 to 5): 2.424 and 1.372 for the original and extended Mixed Influence Model respectively; 1.770 and 1.451 for the original and extended Asymmetric Influence Model respectively; 1.290 and 0.781 for the original and extended Karmeshu-Goswami Model respectively. Note that this is not a measure of overall fit for the extended models, as the aggregate penetration data are only a subset of the data used to calibrate these models.

[INSERT TABLE 2 AND FIGURE 2 ABOUT HERE]

Study 1 has established the feasibility of combining social interactions data with penetration data to calibrate the extended models described in Section 2. It has also provided some evidence that the incorporation of social interactions data gives rise to improved diffusion forecasts. We conducted a second study to replicate these findings in a different context. In particular, Study 2 was conducted in a different geographical market (USA versus Israel), with a different product category (oral care versus feminine hygiene), and in a different time frame (4-week periods versus 12-week periods). Moreover, the social interactions data in Study 2 came from a sample of consumers who had not all received the same number of recommendations (in contrast to Study 1 in which all consumers in the sample had received one recommendation).

b. Study 2

Set up and data

Study 2 was conducted in collaboration with a major US-based CPG manufacturer. The company was interested in the penetration of a new oral care product launched by one of its competitors. This product offered a significantly new benefit and represented an innovation in that category. For confidentiality reasons, we will refer to this new product as PROD2.

We define one time period as 4 weeks. Our social interactions data came from a survey conducted at t=5. The survey was administered online through a professional market research company. The respondents were 1,239 members of a representative panel of the target market. Our survey started with a set of screening questions designed to ensure the quality of the responses. We presented respondents with a list of oral care brands and asked them to indicate which ones they were aware of. The list included a set of fictitious brands, and we removed from the analysis all respondents who indicated that they were aware of at least one fictitious brand. After this screening, we were left with N = 584 respondents. The very high proportion of respondents screened out, despite the fact that the survey was performed by a professional market research company, suggests that great care should be taken to ensure the quality of online data.

The heart of the survey consisted of the following three questions:

- "How many consumers have recommended PROD2 to you?" (numerical answer). Let r_i be the number entered by respondent i, which in our data varied between 0 and 3.
- "Have you purchased PROD2 before?" (Yes/No). Let y_i=1 if respondent i answered "yes" to that question, and 0 otherwise. A total of 53 respondents answered "yes" to that question.

- If y_i=1: "To how many people have you recommended PROD2 so far?" (numerical answer). Let d_i be the number entered by respondent i, which in our data varied between 0 and 8.

In addition to these data, we received aggregate penetration data from a professional market research company, for periods t=1 to t=11. We used t=1…5 as calibration data and t=6…11 as validation data.

Calibration

Calibration followed an approach similar to Study 1. One important difference is that in Study 2, our sample of consumers had not all received the same number of recommendations. As a result, in Study 2 we can use both the aggregate penetration data and the survey adoption data (y_i) to calibrate the original Mixed Influence Model, Asymmetric Influence Model and Karmeshu-Goswami Model. In addition, the social interactions data used to calibrate the extended models now include the number of recommendations received, r_i.

We describe here the calibration of the extended and original Mixed Influence Model (starting with the extended model). The likelihood contribution of each piece of social interactions data in the other models may be found in Appendix 3 (extended Asymmetric Influence Model) and Appendix 4 (Karmeshu-Goswami Model).

Our social interactions data in Study 2 consist of r_i (number of recommendations received), y_i (adoption), and for those consumers who adopted, d_i (number of recommendations given). The likelihood for the social interactions data may be decomposed as follows:

$$P^{EXT\text{-}MIM}(y_i,d_i,r_i)= P^{EXT\text{-}MIM}(d_i|y_i,r_i).\ P^{EXT\text{-}MIM}(y_i|r_i).\ P^{EXT\text{-}MIM}(r_i)$$

Let us first consider $P^{EXT\text{-}MIM}(r_i)$, the probability of receiving r_i recommendations over the first 5 periods. Let r_{it} refer to the number of recommendations received by consumer i in period t. $P^{EXT\text{-}MIM}(r_i)$ is obtained by summing over all possible patterns of recommendations $\{r_{i1}, r_{i2}, \ldots r_{i5}\}$ where $r_{i1}+r_{i2}+\ldots+r_{i5}=r_i$:

$$P^{EXT-MIM}(r_i) = \sum_{\{r_{it}\}_{t=1,..5}|r_{i1}+..,r_{i5}=r_i} P^{EXT-MIM}(\{r_{it}\}_{t=1,..5}) = \sum_{\{r_{it}\}_{t=1,..5}|r_{i1}+..,r_{i5}=r_i} \prod_{t=1}^{5} \binom{ties}{r_{it}} .(a.F_{t-1})^{r_{it}} .(1-a.F_{t-1})^{ties-r_{it}} \quad (17)$$

Let us next consider $P(y_i|r_i)$, which captures the probability of adopting over the first 5 periods conditional on having received r_i recommendations during that time. Based on the conditional hazard rate from Equation (1), the cumulative hazard rate conditional on r_i is:

$$P^{EXT-MIM}(y_i = 1 | r_i) = 1-(1-p)^5(1-q)^{r_i} \quad (18)$$

We next turn to the specification of $P(d_i|y_i,r_i)$, which captures the likelihood contribution of the number of recommendations given, d_i. We consider only $P(d_i|y_i=1,r_i)$, as d_i is only available for respondents for whom $y_i=1$. Similarly to Study 1, we have:

$$P^{EXT-MIM}(d_i | y_i = 1, r_i) = \sum_{t_i=1}^{5} \binom{ties.(5-t_i)}{d_i} .(1-a)^{(ties.(5-t_i)-d_i)} .a^{d_i} .P^{EXT-MIM}(adopted\ in\ t_i | y_i = 1, r_i) \quad (19)$$

where: $P^{EXT-MIM}(adopted\ in\ t_i | y_i = 1, r_i) = \dfrac{P^{EXT-MIM}(adopted\ in\ t_i | r_i)}{\sum_{t=1}^{T} P^{EXT-MIM}(adopted\ in\ t | r_i)}$.

The probability $P^{EXT-MIM}(adopted\ in\ t_i | r_i)$ is obtained by summing over all possible patterns of recommendations:

$$P^{EXT-MIM}(adopted\ in\ t_i \mid r_i) = \sum_{\{r_{it}\}_{t=1,..5} \mid r_{i1}+..,r_{i5}=r_i} P^{EXT-MIM}(adopted\ in\ t_i, \{r_{it}\}_{t=1,..5} \mid r_i)$$

$$= \sum_{\{r_{it}\}_{t=1,..5} \mid r_{i1}+..,r_{i5}=r_i} P^{EXT-MIM}(adopted\ in\ t_i \mid \{r_{it}\}_{t=1,..5}).P(\{r_{it}\}_{t=1,..5} \mid r_i) \qquad (20)$$

$$= \frac{\sum_{\{r_{it}\}_{t=1...5} \mid r_{i1}+..,r_{i5}=r_i} (1-p)^{t_i-1}.(1-q)^{r_{i1}+..r_{it_i-1}}.(1-(1-p).(1-q)^{r_{it_i}}).P^{EXT-MIM}(\{r_{it}\}_{t=1...5})}{\sum_{\{r_{it}\}_{t=1...5} \mid r_{i1}+..,r_{i5}=r_i} P^{EXT-MIM}(\{r_{it}\})}$$

where $P^{EXT-MIM}(\{r_{it}\}_{t=1...5})$ is given in Equation (17).

Finally, the aggregate penetration data is modeled as in Study 1:

$$S_t = m.f_t^{EXT-MIM} + \varepsilon_t, \qquad \varepsilon_t \sim N(0,\sigma^2)$$

where $f_t^{EXT\text{-}MIM}$ is obtained by applying Equation (3) recursively.

We use a similar Bayesian MCMC procedure as in Study 1, with the same diffuse prior.

The original Mixed Influence Model was estimated similarly to Study 1, with the important exception that the survey adoption data was combined with the aggregate penetration data:

$$S_t = m.f_t^{MIM} + \varepsilon_t \qquad \varepsilon_t \sim N(0,\sigma^2)$$

$$P^{MIM}(y_i) = F_5^{MIM}$$

Out-of-sample predictive performance

Out-of-sample predictive ability was assessed similarly as in Study 1. The results are reported in Table 3, with the posterior distributions of the total cumulative proportion of adopters over the validation periods plotted in Figure 3. As in Study 1, the out-of-sample predictive ability corresponding to the extended version of each model is lower than that corresponding to the original model, and Figure 3 suggests that the improvement is at least partly driven by a lower variance of the predictions. The true value is always contained in the 95% credible interval for each extended and original model. However the *width* of the 95% credible interval obtained from the extended Mixed Influence Model (respectively, the Asymmetric Influence Model and the Karmeshu-Goswami Model) is 64% (respectively 32% and 56%) of the width of the 95% credible interval obtained from the original model. Note that the mean squared error between the true and predictive out-of-sample penetration is substantially larger in Study 2 compared to Study 1, for all models. Further analysis suggests that this is primarily driven by the fact that the number of validation periods is greater in Study 2 compared to Study 1.

Although our focus is on diffusion forecasts, as in Study 1 we also report the mean squared error between the true and predicted cumulative number of adopters in the *calibration* periods (t=1 to 5): 11.432 and 12.368 for the original and extended Mixed Influence Model respectively; 11.209 and 11.381 for the original and extended Asymmetric Influence Model respectively; 11.432 and 11.593 for the original and extended Karmeshu-Goswami Model respectively. While the measures are comparable for all models, the mean squared error for each extended model is slightly higher compared to the original model. As noted above, these measures are not measures of the overall fit of the models, because the aggregate penetration data are only a subset of the calibration data.

[INSERT TABLE 3 AND FIGURE 3 ABOUT HERE]

4. Conclusions and Directions for Future Research

We have proposed an approach for using social interactions data, which have become increasingly available in the past few years, to improve the diffusion forecasts made by major extant diffusion models. In order to accommodate these data, we have extended major diffusion models to capture explicitly the generation of social interactions and their impact on adoption. Empirically, combining social interactions data with penetration data appears to give rise to improved diffusion forecasts. The use of social interactions data does not preclude the use of other ancillary data suggested in previous research (e.g, analogous past innovations, timing of adoption).

We believe that several areas for future research may be identified. First, while our focus here has been on extending existing models, the framework proposed in this paper may be used to develop new models that make sets of assumptions different from those made by extant models. For example, heterogeneity in the number of social ties may be captured (Barabási and Albert 1999). Second, the framework itself may be extended for example to capture negative word-of-mouth (Mahajan, Muller and Kerin, 1984), specific network structures (Shaikh, Rangaswamy, and Balakrishnan, 2007; Watts and Strogatz 1998), different types of ties or relationships (Ansari, Koenigsberg and Stahl 2008; Iyengar, Van den Bulte and Valente 2009), or to include covariates such as marketing mix variables (Bass, Krishnan and Jain, 1994; Horsky and Simon, 1983; Kalish and Sen, 1986; Robinson and Lakhani, 1975). Finally, the proposed models use discrete time intervals, making the parameters a function of the data frequency. Future research may explore continuous-time versions.

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Tables and Figures

Name of variable in extended Mixed Influence Model	Similar variables in other extended models	Definition
p	p^{1} , p^{2} ,p^{low}, p^{high}	Captures the effect of external forces on adoption
q	q^{1} ,q^{2} ,q^{low} ,q^{high}	Probability of adopting based on one recommendation
n_t	n_t^{1-1} , n_t^{1-2} , n_t^{2-2}	Number of recommendations received by a potential adopter in period *t*
ties	$ties^{1-2}$,$ties^{1-1}$,$ties^{2-2}$	Number of social ties
a	a^{1-1}, a^{1-2}, a^{2-2}	Probability that an adopter recommends the innovation to each of his or her social ties in each period
f_t	f_t^{1} ,f_t^{2} ,$f_t^{low\text{-}low}$,$f_t^{high\text{-}low}$,$f_t^{low\text{-}high}$,$f_t^{high\text{-}high}$	Marginal penetration in period *t*
F_t	F_t^{1} ,F_t^{2} ,$F_t^{low\text{-}low}$,$F_t^{high\text{-}low}$,$F_t^{low\text{-}high}$,$F_t^{high\text{-}high}$	Cumulative penetration by the end of period *t*

Table 1: list of variables.

	Out-of-sample predictive ability: True versus predicted total number of adopters in holdout periods	
Model	**Original**	**Extended**
Mixed Influence Model	2.177	0.318
Asymmetric Influence Model	0.677	0.468
Karmeshu-Goswami Model	3.043	1.705

Table 2: Out-of-sample predictive ability – Study 1.
Note: Reported numbers are the mean squared errors (over posterior draws) of predicted number of adopters (expressed in percent of target market) versus actual.

	Out-of-sample predictive ability: True versus predicted total number of adopters in holdout periods	
Model	**Original**	**Extended**
Mixed Influence Model	105.561	56.321
Asymmetric Influence Model	85.884	8.447
Karmeshu-Goswami Model	64.854	23.985

Table 3: Out-of-sample predictive ability – Study 2.
Note: Reported numbers are the mean squared errors (over posterior draws) of predicted number of adopters (expressed in percent of target market) versus actual.

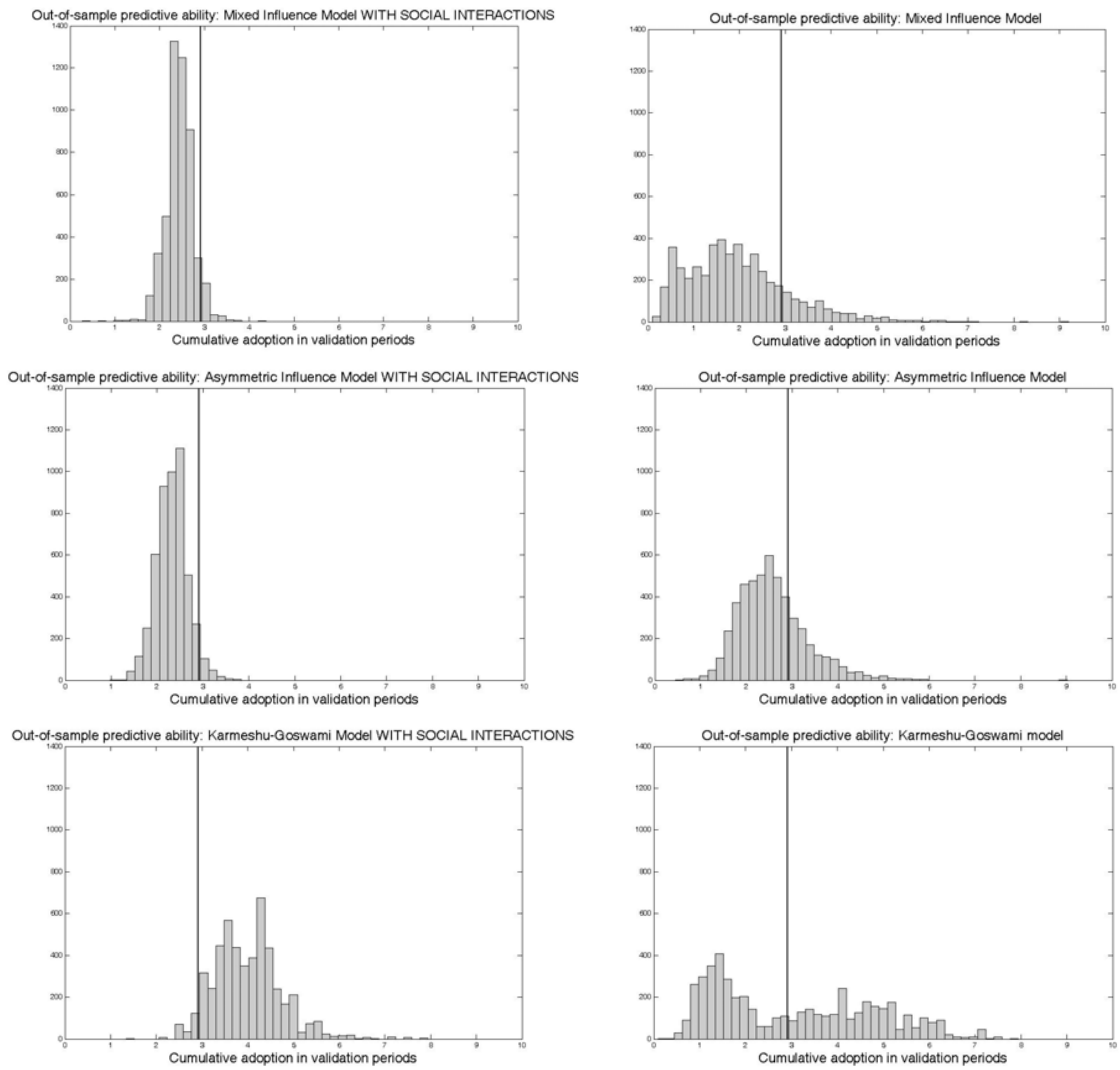

Figure 2. Posterior checks for out-of-sample predictive ability – Study 1.
Note: the histograms show the posterior distributions of the total aggregate number of adopters in the holdout validation periods (expressed in percent of the target market), the solid line represents the actual value.

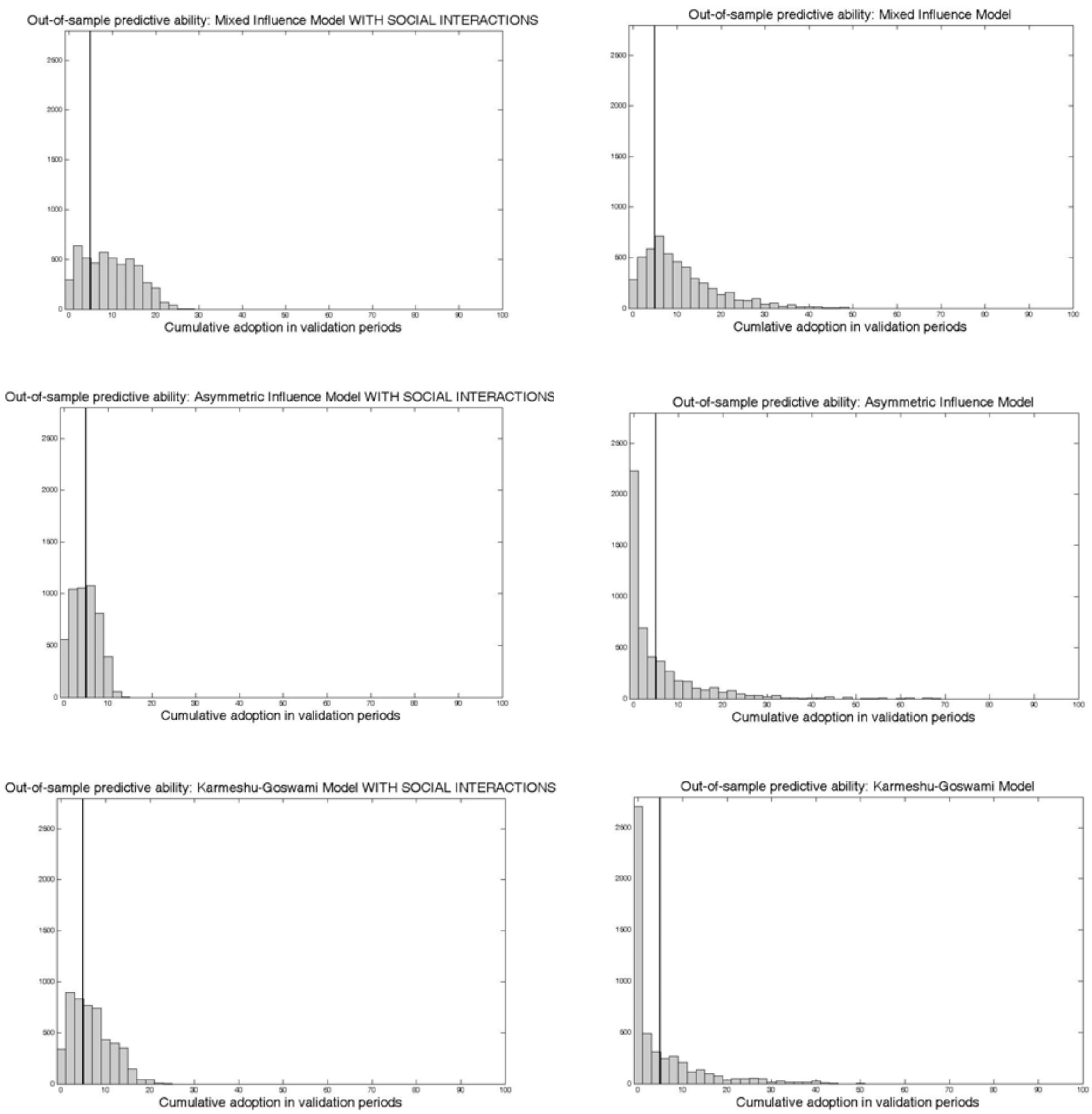

Figure 3. Posterior checks for out-of-sample predictive ability – Study 2.
Note: the histograms show the posterior distributions of the total aggregate number of adopters in the holdout validation periods (expressed in percent of the target market), the solid line represents the actual value.

Appendix 1: Calibration of the extended Asymmetric Influence Model – Study 1

We describe here the likelihood contribution of each piece of social interactions data. We refer to r_{it} as the number of recommendations received by consumer *i* during period *t*.

Adoption conditional on receiving one recommendation in t=5 ($P(y_i|\{r_{i1},.. r_{i5}\}=\{0,0,0,0,1\})$):

$$P^{EXT-AIM}(y_i = 1 | \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\})$$

$$= [1-(1-p_1)^5(1-q_1)].P^{EXT-AIM}(innovator | \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\})$$

$$+[1-(1-p_2)^5(1-q_2)].P^{EXT-AIM}(imitator | \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\})$$

$$P^{EXT-AIM}(innovator | \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\})$$

$$= \frac{P^{EXT-AIM}(\{r_{it}\}_{t=1..5} = \{0,0,0,0,1\} | innovator).\theta}{P^{EXT-AIM}(\{r_{it}\}_{t=1..5} = \{0,0,0,0,1\} | innovator).\theta + P^{EXT-AIM}(\{r_{it}\}_{t=1..5} = \{0,0,0,0,1\} | imitator).(1-\theta)}$$

$$P^{EXT-AIM}(imitator | \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\}) = 1 - P^{EXT-AIM}(innovator | \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\})$$

Special case a):

$$P^{EXT-AIM}(\{r_{it}\}_{t=1..5} = \{0,0,0,0,1\} | innovator)$$

$$= [(1-a^{1-1}.F_1^1).(1-a^{1-1}.F_2^1).(1-a^{1-1}.F_3^1)]^{ties^{1-1}}.ties^{1-1}.(a^{1-1}F_4^1).(1-a^{1-1}F_4^1)^{ties^{1-1}-1}$$

$$P^{EXT-AIM}(\{r_{it}\}_{t=1..5} = \{0,0,0,0,1\} | imitator)$$

$$= [(1-a^{2-2}F_1^2).(1-a^{2-2}F_2^2).(1-a^{2-2}F_3^2)]^{ties^{2-2}}.ties^{2-2}.(a^{2-2}F_4^2).(1-a^{2-2}F_4^2)^{ties^{2-2}-1}$$

Special case b):

$P^{EXT-AIM}(\{r_{it}\}_{t=1..5} = \{0,0,0,0,1\} | innovator) = 0$ and therefore

$$P^{EXT-AIM}(innovator | \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\}) = 0$$

Number of recommendations given conditional on adoption and recommendations received

($P(d_i|y_i=1,\{r_{i1},..\ r_{i5}\}=\{0,0,0,0,1\})$):

$$P^{EXT-MIM}(d_i \mid y_i=1,\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\})$$
$$=P^{EXT-MIM}(d_i \mid y_i=1,\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\},innovator).P^{EXT-AIM}(innovator \mid y_i=1,\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\})$$
$$+P^{EXT-MIM}(d_i \mid y_i=1,\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\},imitator).P^{EXT-AIM}(imitator \mid y_i=1,\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\})$$

$$P^{EXT-AIM}(innovator \mid y_i=1,\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\})$$
$$=\frac{P^{EXT-AIM}(\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\},y_i=1 \mid innovator).\theta}{P^{EXT-AIM}(\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\},y_i=1 \mid innovator).\theta+P^{EXT-AIM}(\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\},y_i=1 \mid imitator).(1-\theta)}$$

$$P^{EXT-AIM}(\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\},y_i=1 \mid innovator)=P^{EXT-AIM}(\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\} \mid innovator).(1-(1-p_1)^5.(1-q_1))$$
$$P^{EXT-AIM}(\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\},y_i=1 \mid imitator)=P^{EXT-AIM}(\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\} \mid imitator).(1-(1-p_2)^5.(1-q_2))$$

Special case a):

$$P^{EXT-AIM}(d_i \mid y_i=1,\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\},innovator)$$
$$=\sum_{t_i=1}^{5}\begin{pmatrix} ties^{1-1}.(5-t_i) \\ d_i \end{pmatrix}.(1-a^{1-1})^{(ties^{1-1}.(5-t_i)-d_i)}.a^{1-1^{d_i}}.P^{EXT-AIM}(adopted\ in\ t_i \mid y_i=1,\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\},innovator)$$

where:

$$P^{EXT-MIM}(adopted\ in\ t_i \mid y_i=1,\{r_{it}\}_{t=1..5}=\{0,0,0,0,1\},innovator)$$
$$=\frac{P^{EXT-MIM}(adopted\ in\ t_i \mid \{r_{it}\}_{t=1..5}=\{0,0,0,0,1\},innovator)}{\sum_{t=1}^{5}P^{EXT-MIM}(adopted\ in\ t \mid \{r_{it}\}_{t=1..5}=\{0,0,0,0,1\},innovator)}$$

and $P^{EXT-MIM}(adopted\ in\ t_i \mid \{r_{it}\}_{t=1..5}=\{0,0,0,0,1\},innovator)=(1-p_1)^{t_i-1}.(1-(1-p_1).(1-q_1)^{1(t_i=5)})$

$$P^{EXT-AIM}(d_i \mid y_i = 1, \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\}, imitator)$$

$$= \sum_{t_i=1}^{5} \binom{ties^{2-2}.(5-t_i)}{d_i} .(1-a^{2-2})^{(ties^{2-2}.(5-t_i)-d_i)} .a^{2-2^{d_i}} .P^{EXT-AIM}(adopted\ in\ t_i \mid y_i = 1, \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\}, imitator)$$

where:

$$P^{EXT-MIM}(adopted\ in\ t_i \mid y_i = 1, \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\}, imitator)$$

$$= \frac{P^{EXT-MIM}(adopted\ in\ t_i \mid \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\}, imitator)}{\sum_{t=1}^{5} P^{EXT-MIM}(adopted\ in\ t \mid \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\}, imitator)}$$

and $P^{EXT-MIM}(adopted\ in\ t_i \mid \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\}, imitator) = (1-p_2)^{t_i-1}.(1-(1-p_2).(1-q_2)^{1(t_i=5)})$

<u>Special case b):</u>

$P^{EXT-AIM}(innovator \mid y_i = 1, \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\}) = 0$ and therefore:

$$P^{EXT-AIM}(d_i \mid y_i = 1, \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\}) = P^{EXT-AIM}(d_i \mid y_i = 1, \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\}, imitator)$$

where $P^{EXT-AIM}(d_i \mid y_i = 1, \{r_{it}\}_{t=1..5} = \{0,0,0,0,1\}, imitator)$ is as in special case a).

Appendix 2: Calibration of the extended Karmeshu-Goswami Model – Study 1

We describe here the likelihood contribution of each piece of social interactions data. We refer to r_{it} as the number of recommendations received by consumer i during period t.

Adoption conditional on receiving one recommendation in t=5 ($P(y_i|\{r_{i1},.. r_{i5}\}=\{0,0,0,0,1\})$):

$$P^{EXT-KG}(y_i = 1 \mid \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\})$$

$$= [1-(1-p_{low})^5(1-q_{low})].P^{EXT-KG}(p_{low}, q_{low} \mid \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\})$$

$$+[1-(1-p_{low})^5(1-q_{high})].P^{EXT-KG}(p_{low}, q_{high} \mid \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\})$$

$$+[1-(1-p_{high})^5(1-q_{low})].P^{EXT-KG}(p_{high}, q_{low} \mid \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\})$$

$$+[1-(1-p_{high})^5(1-q_{high})].P^{EXT-KG}(p_{high}, q_{high} \mid \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\})$$

$$P^{EXT-KG}(p_{low}, q_{low} \mid \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\}) = \frac{P^{EXT-KG}(\{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\} \mid p_{low}, q_{low}).g(p_{low}, q_{low})}{\sum_{p\in\{p_{low},p_{high}\},q\in\{q_{low},q_{high}\}} P^{EXT-KG}(\{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\} \mid p,q).g(p,q)}$$

$$P^{EXT-KG}(\{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\} \mid p_{low}, q_{low})$$

$$= [(1-a.F_1^{low-low}).(1-a.F_2^{low-low}).(1-a.F_3^{low-low})]^{ties}.ties.a.F_4^{low-low}.(1-a.F_4^{low-low})^{ties-1}$$

Number of recommendations given conditional on adoption and recommendations received ($P(d_i|y_i=1,\{r_{i1},.. r_{i5}\}=\{0,0,0,0,1\})$):

$$P^{EXT-KG}(d_i \mid y_i = 1, \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\})$$

$$= \sum_{p\in\{p_{low},p_{high}\},q\in\{q_{low},q_{high}\}} P^{EXT-KG}(d_i \mid y_i = 1, \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\}, p, q).P^{EXT-KG}(p,q \mid y_i = 1, \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\})$$

$$P^{EXT-KG}(p,q \mid y_i = 1, \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\})$$

$$= \frac{P^{EXT-KG}(\{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\}, y_i = 1 \mid p,q).g(p,q)}{\sum_{p' \in \{p_{low}, q_{high}\}, q' \in \{q_{low}, q_{high}\}} P^{EXT-KG}(\{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\}, y_i = 1 \mid p',q').g(p',q')}$$

where

$$P^{EXT-KG}(\{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\}, y_i = 1 \mid p,q) = P^{EXT-KG}(\{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\} \mid p,q).(1-(1-p)^5.(1-q))$$

$$P^{EXT-AIM}(d_i \mid y_i = 1, \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\}, p, q)$$

$$= \sum_{t_i=1}^{5} \begin{pmatrix} ties.(5-t_i) \\ d_i \end{pmatrix}.(1-a)^{(ties.(5-t_i)-d_i)}.a^{d_i}.P^{EXT-KG}(adopted\ in\ t_i \mid y_i = 1, r_i, p, q)$$

where

$$P^{EXT-KG}(adopted\ in\ t_i \mid y_i = 1, \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\}, p, q)$$

$$= \frac{P^{EXT-KG}(adopted\ in\ t_i \mid \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\}, p, q)}{\sum_{t=1}^{5} P^{EXT-KG}(adopted\ in\ t \mid \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\}, p, q)}$$

and $P^{EXT-KG}(adopted\ in\ t_i \mid \{r_{it}\}_{t=1,..5} = \{0,0,0,0,1\}, p, q) = (1-p)^{t_i - 1}.(1-(1-p).(1-q)^{1(t_i=5)})$

Appendix 3: Calibration of the extended Asymmetric Influence Model – Study 2

We describe here the likelihood contribution of each piece of social interactions data. We refer to r_{it} as the number of recommendations received by consumer i during period t.

<u>Number of recommendations received ($P(r_i)$):</u>

$$P^{EXT-AIM}(r_i) = P^{EXT-AIM}(r_i \mid innovator).\theta + P^{EXT-AIM}(r_i \mid imitator).(1-\theta)$$

Special case a):

$$P^{EXT-AIM}(r_i \mid innovator) = \sum_{\{r_{it}\}_{t=1,..5} \mid r_{i1}+..,r_{i5}=r_i} P^{EXT-AIM}(\{r_{it}\}_{t=1,..T} \mid innovator)$$

$$= \sum_{\{r_{it}\}_{t=1,..5} \mid r_{i1}+..,r_{i5}=r_i} \prod_{t=1}^{5} \binom{ties^{1-1}}{r_{it}} .(a^{1-1}.F_{t-1}^{1})^{r_{it}}.(1-a^{1-1}.F_{t-1}^{1})^{ties-r_{it}}$$

$$P^{EXT-AIM}(r_i \mid imitator) = \sum_{\{r_{it}\}_{t=1,..5} \mid r_{i1}+..,r_{i5}=r_i} P^{EXT-AIM}(\{r_{it}\}_{t=1,..T} \mid imitator)$$

$$= \sum_{\{r_{it}\}_{t=1,..5} \mid r_{i1}+..,r_{i5T}=r_i} \prod_{t=1}^{5} \binom{ties^{2-2}}{r_{it}} .(a^{2-2}.F_{t-1}^{2})^{r_{it}}.(1-a^{2-2}.F_{t-1}^{2})^{ties-r_{it}}$$

Special case b):

$$P^{EXT-AIM}(r_i \mid innovator) = 1(r_i = 0)$$

$$P^{EXT-AIM}(r_i \mid imitator) = \sum_{\{r_{it}^{1-2}, r_{it}^{2-2}\}_{t=1,..5} \mid r_{i1}^{1-2}+r_{i1}^{2-2}+..+r_{i5}^{1-2}+r_{i5}^{2-2}=r_i} P^{EXT-AIM}(\{r_{it}^{1-2}, r_{it}^{2-2}\}_{t=1,..5} \mid imitator)$$

where:

$$P^{EXT-AIM}(\{r_{it}^{1-2}, r_{it}^{2-2}\}_{t=1,..5} \mid imitator)$$

$$= \prod_{t=1}^{5} \binom{ties^{1-2}}{r_{it}^{1-2}}.(a^{1-2}.F_{t-1}^{1})^{r_{it}^{1-2}}.(1-a^{1-2}.F_{t-1}^{1})^{ties^{1-2}-r_{it}^{1-2}}.\binom{ties^{2-2}}{r_{it}^{2-2}}.(a^{2-2}.F_{t-1}^{2})^{r_{it}^{2-2}}.(1-a^{2-2}.F_{t-1}^{2})^{ties^{2-2}-r_{it}^{2-2}}$$

<u>Adoption conditional on recommendations received ($P(y_i|r_i)$):</u>

$$P^{EXT-AIM}(y_i = 1 \mid r_i)$$

$$= [1-(1-p_1)^5(1-q_1)^{r_i}].P^{EXT-AIM}(innovator \mid r_i) + [1-(1-p_2)^5(1-q_2)^{r_i}].P^{EXT-AIM}(imitator \mid r_i)$$

$$P^{EXT-AIM}(innovator \mid r_i) = \frac{P^{EXT-AIM}(r_i \mid innovator).\theta}{P^{EXT-AIM}(r_i \mid innovator).\theta + P^{EXT-AIM}(r_i \mid imitator).(1-\theta)}$$

<u>Number of recommendations given conditional on adoption and recommendations received</u>

<u>($P(d_i|y_i=1,r_i)$):</u>

$$P^{EXT-MIM}(d_i \mid y_i = 1, r_i)$$

$$= P^{EXT-MIM}(d_i \mid y_i = 1, r_i, innovator).P^{EXT-AIM}(innovator \mid y_i = 1, r_i)$$

$$+ P^{EXT-MIM}(d_i \mid y_i = 1, r_i, imitator).P^{EXT-AIM}(imitator \mid y_i = 1, r_i)$$

$$P^{EXT-AIM}(innovator \mid y_i = 1, r_i) = \frac{P^{EXT-AIM}(r_i, y_i = 1 \mid innovator).\theta}{P^{EXT-AIM}(r_i, y_i = 1 \mid innovator).\theta + P^{EXT-AIM}(r_i, y_i = 1 \mid imitator).(1-\theta)}$$

$$P^{EXT-AIM}(r_i, y_i = 1 \mid innovator) = P^{EXT-AIM}(r_i \mid innovator).(1-(1-p_1)^5.(1-q_1)^{r_i})$$

$$P^{EXT-AIM}(r_i, y_i = 1 \mid imitator) = P^{EXT-AIM}(r_i \mid imitator).(1-(1-p_2)^5.(1-q_2)^{r_i})$$

Special case a):

$$P^{EXT-AIM}(d_i \mid y_i = 1, r_i, innovator)$$

$$= \sum_{t_i=1}^{5} \binom{ties^{1-1}.(5-t_i)}{d_i}.(1-a^{1-1})^{(ties^{1-1}.(5-t_i)-d_i)}.a^{1-1^{d_i}}.P^{EXT-AIM}(adopted\ in\ t_i \mid y_i = 1, r_i, innovator)$$

where:

$$P^{EXT-MIM}(adopted\ in\ t_i \mid y_i = 1, r_i, innovator) = \frac{P^{EXT-MIM}(adopted\ in\ t_i \mid r_i, innovator)}{\sum_{t=1}^{5} P^{EXT-MIM}(adopted\ in\ t \mid r_i, innovator)}$$

and:

$$P^{EXT-MIM}(adopted\ in\ t_i \mid r_i, innovator)$$

$$= \frac{\sum_{\{r_{it}\}_{t=1...5} | r_{i1}+..,r_{i5}=r_i} (1-p_1)^{t_i-1}.(1-q_1)^{r_{i1}+..r_{it_i-1}}.(1-(1-p_1).(1-q_1)^{r_{it_i}}).P^{EXT-MIM}(\{r_{it}\} \mid innovator)}{\sum_{\{r_{it}\}_{t=1...5} | r_{i1}+..,r_{i5}=r} P^{EXT-MIM}(\{r_{it}\} \mid innovator)}$$

$$P^{EXT-AIM}(d_i \mid y_i = 1, r_i, imitator)$$

$$= \sum_{t_i=1}^{5} \binom{ties^{2-2}.(5-t_i)}{d_i}.(1-a^{2-2})^{(ties^{2-2}.(5-t_i)-d_i)}.a^{2-2^{d_i}}.P^{EXT-AIM}(adopted\ in\ t_i \mid y_i = 1, r_i, imitator)$$

where:

$$P^{EXT-MIM}(adopted\ in\ t_i \mid y_i = 1, r_i, imitator) = \frac{P^{EXT-MIM}(adopted\ in\ t_i \mid r_i, imitator)}{\sum_{t=1}^{5} P^{EXT-MIM}(adopted\ in\ t \mid r_i, imitator)}$$

and:

$$P^{EXT-MIM}(adopted\ in\ t_i \mid r_i, imitator)$$

$$= \frac{\sum_{\{r_{it}\}_{t=1...5} | r_{i1}+..,r_{iT}=r_i} (1-p_2)^{t_i-1}.(1-q_2)^{r_{i1}+..r_{it_i-1}}.(1-(1-p_2).(1-q_2)^{r_{it_i}}).P^{EXT-MIM}(\{r_{it}\} \mid imitator)}{\sum_{\{r_{it}\}_{t=1...5} | r_{i1}+..,r_{i5}=r} P^{EXT-MIM}(\{r_{it}\} \mid imitator)}$$

Special case b):

$$P^{EXT-AIM}(d_i \mid y_i = 1, r_i, innovator)$$

$$= \sum_{t_i=1}^{5} \begin{pmatrix} ties^{1-2}.(5-t_i) \\ d_i \end{pmatrix} .(1-a^{1-2})^{(ties^{1-2}.(5-t_i)-d_i)}.a^{1-2^{d_i}}.P^{EXT-AIM}(adopted\ in\ t_i \mid y_i = 1, r_i, innovator)$$

where:

$$P^{EXT-MIM}(adopted\ in\ t_i \mid y_i = 1, r_i, innovator) = \frac{P^{EXT-MIM}(adopted\ in\ t_i \mid r_i, innovator)}{\sum_{t=1}^{5} P^{EXT-MIM}(adopted\ in\ t \mid r_i, innovator)}$$

$$P^{EXT-MIM}(adopted\ in\ t_i \mid r_i, innovator) = (1-p_1)^{t_i-1} p_1$$

$$P^{EXT-AIM}(d_i \mid y_i = 1, r_i, imitator)$$

$$= \sum_{t_i=1}^{5} \begin{pmatrix} ties^{2-2}.(5-t_i) \\ d_i \end{pmatrix} .(1-a^{2-2})^{(ties^{2-2}.(5-t_i)-d_i)}.a^{2-2^{d_i}}.P^{EXT-AIM}(adopted\ in\ t_i \mid y_i = 1, r_i, imitator)$$

where

$$P^{EXT-MIM}(adopted\ in\ t_i \mid y_i = 1, r_i, imitator) = \frac{P^{EXT-MIM}(adopted\ in\ t_i \mid r_i, imitator)}{\sum_{t=1}^{5} P^{EXT-MIM}(adopted\ in\ t \mid r_i, imitator)}$$

and:

$$P^{EXT-MIM}(adopted\ in\ t_i \mid r_i, imitator)$$

$$= \frac{\sum_{\{r_{it}^{1-2}, r_{it}^{2-2}\}_{t=1,..5} \mid r_{i1}^{1-2}+r_{i1}^{2-2}+..+r_{i5}^{1-2}+r_{i5}^{2-2}=r_i} (1-p_2)^{t_i-1}.(1-q_2)^{r_{i1}^{1-2}+r_{i1}^{2-2}+..r_{it_i-1}^{1-2}+r_{it_i-1}^{2-2}}.(1-(1-p_2).(1-q_2)^{r_{it_i}^{1-2}+r_{it_i}^{2-2}}).P^{EXT-AIM}(\{r_{it}^{1-2}, r_{it}^{2-2}\}_{t=1,..5} \mid imitator)}{\sum_{\{r_{it}^{1-2}, r_{it}^{2-2}\}_{t=1,..5} \mid r_{i1}^{1-2}+r_{i1}^{2-2}+..+r_{i5}^{1-2}+r_{i5}^{2-2}=r_i} P^{EXT-AIM}(\{r_{it}^{1-2}, r_{it}^{2-2}\}_{t=1,..T} \mid imitator)}$$

Appendix 4: Calibration of the extended Karmeshu-Goswami Model – Study 2

We describe here the likelihood contribution of each piece of social interactions data. We refer to r_{it} as the number of recommendations received by consumer *i* during period *t*.

Number of recommendations received ($P(r_i)$):

$$P^{EXT-KG}(r_i) = \sum_{p \in \{p_{low}, p_{high}\}, q \in \{q_{low}, q_{high}\}} P^{EXT-KG}(r_i \mid p, q).g(p,q)$$

$$P^{EXT-KG}(r_i \mid p_{low}, q_{low}) = \sum_{\{r_{it}\}_{t=1,..5} | r_{i1}+..,r_{i5}=r_i} P^{EXT-KG}(\{r_{it}\}_{t=1,..5} \mid p_{low}, q_{low}) = \sum_{\{r_{it}\}_{t=1,..5} | r_{i1}+..,r_{i5}=r_i} \prod_{t=1}^{5} \binom{ties}{r_{it}} .(a.F_{t-1}^{low-low})^{r_{it}}.(1-a.F_{t-1}^{low-low})^{ties-r_{it}}$$

And similarly for the other segments.

Adoption conditional on recommendations received ($P(y_i|r_i)$):

$$P^{EXT-KG}(y_i = 1 \mid r_i) = \sum_{p \in \{p_{low}, p_{high}\}, q \in \{q_{low}, q_{high}\}} [1-(1-p)^5(1-q)^{r_i}].P^{EXT-KG}(p,q \mid r_i)$$

where: $$P^{EXT-KG}(p,q \mid r_i) = \frac{P^{EXT-KG}(r_i \mid p,q).g(p,q)}{\sum_{p' \in \{p_{low}, p_{high}\}, q' \in \{q_{low}, q_{high}\}} P^{EXT-KG}(r_i \mid p', q').g(p',q')}$$

Number of recommendations given conditional on adoption and recommendations received ($P(d_i|y_i=1,r_i)$):

$$P^{EXT-KG}(d_i \mid y_i = 1, r_i) = \sum_{p \in \{p_{low}, p_{high}\}, q \in \{q_{low}, q_{high}\}} P^{EXT-KG}(d_i \mid y_i = 1, r_i, p, q).P^{EXT-KG}(p,q \mid y_i = 1, r_i)$$

where: $$P^{EXT-KG}(p,q \mid y_i = 1, r_i) = \frac{P^{EXT-KG}(r_i, y_i = 1 \mid p,q).g(p,q)}{\sum_{p' \in \{p_{low}, q_{high}\}, q' \in \{q_{low}, q_{high}\}} P^{EXT-KG}(r_i, y_i = 1 \mid p', q').g(p',q')}$$

and: $$P^{EXT-KG}(r_i, y_i = 1 \mid p,q) = P^{EXT-KG}(r_i \mid p,q).(1-(1-p)^5.(1-q)^{r_i})$$

$$P^{EXT-AIM}(d_i \mid y_i = 1, r_i, p, q)$$

$$= \sum_{t_i=1}^{5} \binom{ties.(5-t_i)}{d_i} .(1-a)^{(ties.(5-t_i)-d_i)} .a^{d_i} .P^{EXT-KG}(adopted\ in\ t_i \mid y_i = 1, r_i, p, q)$$

where:

$$P^{EXT-KG}(adopted\ in\ t_i \mid y_i = 1, r_i, p, q) = \frac{P^{EXT-KG}(adopted\ in\ t_i \mid r_i, p, q)}{\sum_{t=1}^{T} P^{EXT-KG}(adopted\ in\ t \mid r_i, p, q)}$$

and:

$$P^{EXT-KG}(adopted\ in\ t_i \mid r_i, p, q)$$

$$= \frac{\sum_{\{r_{it}\}_{t=1\ldots5} \mid r_{i1}+\ldots,r_{i5}=r_i} (1-p)^{t_i-1} .(1-q)^{r_{i1}+\ldots r_{it_i-1}} .(1-(1-p).(1-q)^{r_{it_i}}).P^{EXT-KG}(\{r_{it}\} \mid p, q)}{\sum_{\{r_{it}\}_{t=1\ldots5} \mid r_{i1}+\ldots,r_{i5}=r} P^{EXT-KG}(\{r_{it}\} \mid p, q)}$$