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The Effects of Attrition on the Growth and Equity of Competitive Services

Barak Libai, Eitan Muller, and Renana Peres

Customer attrition is an integral part of the growth of new services. This paper presents a diffusion model which explicitly incorporates customer disadoption and interfirm churn dynamics into market growth. A customer equity computation based on the model provides estimations which are notably close to stock market valuations.

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Report Summary

The growth of a new service is similar to a leaking bucket: There is an influx of new customers and, concurrently, an outflow of customers who either switch to competitors or leave the category. This attrition is a major concern for service providers and significantly affects long-range profits.

In this study, the authors investigate the influence of attrition on the growth of service markets. They develop a model of a multifirm growing market, where a firm may acquire customers from the pool of nonusers (which can include new customers as well as customers who disadopted the category in the past) and also acquire customers who switched from competitors. Alternatively, the firm may lose customers who switch or "churn" to a competitor or leave the category entirely. By capturing the complex dynamics of customer acquisition and retention, this model enables an in-depth analysis of the growth of services.

The authors use the model to explore the influence of attrition on the service category and on a particular brand. For service categories, they show that ignoring attrition biases the diffusion parameters and hence affects management diagnostics. For the individual brand, they present a brand-level growth model and use it to capture the effect of attrition on the firm's customer equity: they calculate the customer equity of a growing service and evaluate service firms that operate in competitive industries, including Amazon.com, Barnes&Noble.com, E*Trade, Mobistar, and SK Telecom. For four of the five firms, the results are close to the stock market valuations, which may indicate the role of customer equity in the valuation of growing service firms.

The services growth model adds to the customer equity approach not only by explicitly incorporating customer attrition into market growth, but also by allowing for inter-firm churn dynamics to be included in the estimation. Hence, it is especially well suited to dealing with cases where interfirm customer churn is an integral part of the growth process. ∎

Introduction

Numerous new products introduced into the market during the last few decades are services. Such widely used services as cellular phones and digital TV and financial services such as direct banking were not available before 1980. The growth of the Internet drove the offering of many new services, among them instant messaging, shopping portals, online brokerage, and other online services. Indeed, the service sector in the United States employs most of the workforce, is responsible for more than 80% of the GDP, and is growing considerably faster than the goods sector (Zeithaml and Bitner 2003; BEA 2003).

An important aspect of services that can have considerable influence on the market growth of a new service is customer attrition. Beginning with the initial stages of penetration, there are customers who leave the service: they either switch to competitors or, alternatively, leave the category. In this sense, the growth of a new service is similar to a leaking bucket: there is an inward flow of adopters and a concurrent outward flow of customers who leave.

Customer attrition (or its complement, customer retention) has gained considerable attention from managers and researchers, much of it following demonstrations of the relationship between the sensitivity of firms' long-range profits to changes in its retention rate (Reichheld 1996). Customer retention is a basic component in the computation of customer lifetime value (Kumar and Shah 2004), and its antecedents and consequences for services have been the focus of much research attention in recent years.

Despite these facts, the literature dealing with the evolution of markets for new products has dedicated only sparse efforts to defining and modeling the effect of attrition on the growth of service markets. The diffusion modeling literature, which has been the main thrust of

research effort in this regard (Mahajan, Muller, and Wind 2000; Bass 1969), has generally focused on the growth of category-level markets of single-purchase durable goods and has not been related to customer attrition. Studies examining the growth of competitive markets have generally focused on the competition for acquiring new customers from the remaining market potential and not on interfirm switching (Krishnan, Bass, and Kumar 2000; Kalish, Mahajan, and Muller 1995; Teng and Thompson 1983).

The goal of this paper is to investigate the effects of the dynamics of customer attrition on the growth of markets for services and explore the resultant managerial consequences. We present a multifirm model that captures the complex dynamics of customer acquisition and retention during a service firm's growth: at any period, a firm may acquire customers from the pool of nonusers (which can include new customers as well as customers who disadopted the category in the past) and also acquire customers who switched from competitors. Alternatively, the firm may lose customers who switch or "churn" to a competitor or leave the category entirely. While the dynamics are not trivial, our model is relatively simple and enables an in-depth analysis of the growth of services.

We start with a simpler approach that focuses on category-level growth. This aggregate-level analysis enables us to consider how category-level attrition affects the growth of service categories. We show that neglecting attrition and using the classic diffusion approach—an approach intended originally for durables, yet widely used also for service markets—can create a considerable bias in the parameter estimation and hence in the estimation of growth. We show that disadoption negatively affects not only the lifetime value of a consumer but the effective market potential as well. Thus firms have to invest in reducing disadoption. If the online banking industry, for example, reduces attrition from 16% to 5%, it will gain

over 8 million additional subscribers. While the traditional CRM literature sees the benefits of reduced disadoption in increasing the lifetime value of a single customer through *retention*, our modeling also illustrates the additional gains in terms of *acquisition* of additional customers.

We then use the full model to demonstrate how the combination of category-level and interfirm attrition dynamics affects the calculation of customer equity for service firms. Customer equity, which represents the sum of lifetime values of a firm's customers, has emerged in recent years as a key marketing measure that can be used in assessing the return on marketing activities and the value of firms (Gupta and Lehmann 2005; Peppers and Rogers 2005; Rust, Zeithaml, and Lemon 2000). Recently, Gupta, Lehmann, and Stuart (2004) demonstrated that customer lifetime value and market growth can be combined to estimate the customer equity of a growing firm.

The services growth model adds to the customer equity approach not only by explicitly incorporating customer attrition into market growth, but also by allowing for inter-firm attrition dynamics to be included in the estimation. Hence, it is especially well suited to dealing with cases where interfirm customer churn is an integral part of the growth process, such as mobile cellular services. Similar to Gupta, Lehmann, and Stuart (2004), we compared our customer equity measures with the firm value estimation by the stock market and found that in four out of five cases, our estimations were notably close to the stock market's valuations.

The remainder of this article is organized as follows: We first briefly review the relevant literature concerning attrition and service diffusion. Next, we present our model and its underlying assumptions at the category level and study the influence of disadoption on market growth. We then explore the competi-

tive model and provide a functional form solution for customer equity of a growing service firm and demonstrate its application for firm valuation. We conclude by discussing theoretical and practical implications.

Diffusion and Attrition

Services share common traits with both durable goods and fast-moving consumer goods (FMCG). Similar to FMCG, services greatly rely on repeat purchase for their commercial success. The growth of FMCG is usually attributed to advertising, promotion, and trial; therefore they are usually analyzed using frameworks such as stochastic choice models. In contrast, purchase decision making in services is governed by communication mechanisms such as word-of-mouth and imitation (Wangenhein and Bayon 2004; Murray 1991). In this sense, services are similar to durable goods. However, a major difference between durable goods and services is the outward flow of customers, or customer attrition, wherein a customer decides to terminate the relationship with the provider.

Indeed, the firm-level growth of a service may be viewed as a leaking bucket: On the one hand, there is an inward flow of new customers. These may be nonusers of the service or customers of competitors who just switched suppliers. At the same time, there may be an outward flow of customers due to attrition, some of whom defect to the competitors and some of whom disadopt the service, at least temporarily.

Attrition is mainly relevant to services that entail regular repurchase, where customers develop long-term relationships with the service providers (Berry 1999). Not all service encounters are long term in nature: a one-time dining experience in an out-of-town restaurant does not constitute a relationship. We focus on continuous service encounters that are characterized by some kind of longitudinal cus-

tomer-firm relationship (Bolton and Lemon 1999). Subscription services such as cable TV, phones, online services, Internet service provision, or financial services such as banks and investment houses are good examples of longitudinal relationships.

Attrition (or its complement, retention) has become an important subject when analyzing the relationships of firms with their customers. Since the early 1990s, the business literature has begun to focus on retention rate as a major component of firms' long-term success (Reichheld 1996). In the academic literature, one can see increasing attention on the part of marketing researchers being paid to the antecedents and consequences of customer retention (Lewis 2004; Thomas, Blattberg, and Fox 2004; Lemon, White, and Winer 2002).

Since our approach regards attrition at both the category and firm levels, two research schools might be of interest: the diffusion of innovation modeling and that of competitive dynamics during market growth. Regarding the former, customer attrition has not been formally integrated into the modeling of the diffusion of innovations. The diffusion literature has generally focused on the category level and modeled the diffusion of services as if they were durable goods, including categories such as cellphones (Krishnan, Bass, and Kumar 2000), landline phones (Jain, Mahajan, and Muller 1991), cable TV (Lilien, Rangaswamy, and Van den Bulte 2000), and online banking (Hogan, Lemon, and Libai 2003).

There are diffusion-related studies that have examined cases involving some long-run analysis beyond the original purchase. For example, some research has dealt with the replacement of worn-out units with new ones (Kamakura and Balasubramanian 1987) or multiunit ownership (Steffens 2003), or proposed an integrated model to incorporate the two (Ratchford, Balasubramanian, and Kamakura 2000). The growth of successive generations of products has also been exam-

ined (Mahajan and Muller 1996; Norton and Bass 1987). Focusing on the pharmaceutical industry, the way a new drug is introduced into a competitive market of incumbent brands and the transfer from a nontrial state to trial and regular repeat purchase have been modeled by Hahn et al. (1994) and Lilien, Rao, and Kalish (1981). Yet, in spite of their long-range views, these models have focused on goods and not on services; in particular, they do not deal with the customer-provider relationship, and they do not relate specifically to attrition.

Regarding the competition diffusion literature, some competitive dynamics studies deal with growth under competition (see Chatterjee, Eliashberg, and Rao 2000 for review). They generally investigate one of two scenarios: One scenario is the saturated market that is usually described using a market share formulation. In this scenario, the *total* number of customers remains constant, and the firms compete directly for gaining each other's customers (Chintagunta and Vilcassim 1992). The other scenario, which is usually described using a market potential formulation, is that of a growing market. Models that describe this scenario usually assume that the firms compete for the remaining market potential of non-adopters, but the models do not relate to direct customer transfer between firms (Krishnan, Bass, and Kumar 2000; Givon, Mahajan, and Muller 1995; Kalish, Mahajan, and Muller 1995; Parker and Gatignon 1994; Eliashberg and Jeuland 1986; Teng and Thompson 1983). Thus there is a need for an approach that explicitly incorporates both customer switching and competitive growth.

Types of attrition

In the general term *attrition*, we denote any case of a customer who terminates the relationship with a service provider. Most of the customer profitability literature has assumed that an exiting customer will be acquired by the competition. This kind of attrition is sometimes labeled *churn*. However, in markets

for new services, especially innovative ones, customers may leave the new service category altogether. This type of attrition, which is the type relevant to the growth of the service category, is called *disadoption*. Thus, attrition consists of churning and disadopting customers, and the attrition rate is the sum of the churn rate and the disadoption rate.

Attrition is a major concern in competitive environments. In the mobile telephony industry, cable and satellite TV, e-banking, and other subscriber-based services, attrition is an important operational measurement that is monitored regularly by the service providers. Attrition rates are influenced by customer satisfaction (Bolton 1998), competitive pressure (Oliver 1999), switching costs (Burnham, Frels, and Mahajan 2003), and customer information on the alternatives (Capraro, Broniarczyk, and Srivastava 2003). In competitive industries such as cellular telephony, the churn component of attrition has the attention of many service providers. For example, the introduction of portable numbers, which in many countries considerably increased churn rates, had a substantial effect on the financial performance of the service providers and forced them to drastically change their marketing mix and switching costs structure in order to cope with its consequences (Brown and Drucker 2003).

A growing body of research suggests that in addition to churn, disadoption may also be a substantial problem for marketers, especially in markets using innovative technologies (Hogan, Lemon, and Libai 2003). Empirical evidence for service industries such as online banking, web-based services, and cable TV suggests that many customers stop using the new service during the growth stage (Sarel and Marmorstein 2003; Kramer 2002; Reichheld and Schefter 2000). As innovations become more complex and risky and demand extensive learning, some customers resist them (Ram 1989). This phenomenon has intensified in recent years as consumers often experience

considerable pressure from firms to adopt new services, for example, self-service technologies such as telephone and online response systems (Meuter et al. 2005).
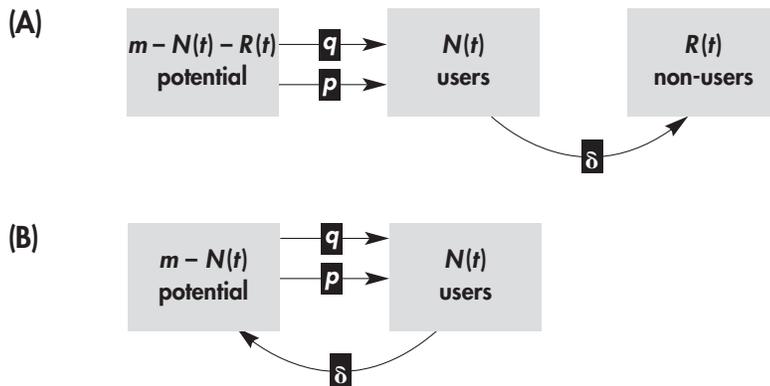
There are two major ways in which marketing modelers have considered customer attrition. *Lost-for-good* attrition occurs when the customer is not expected to come back in the foreseeable future. Due to its simplicity, lost-for-good assumptions have frequently been used for lifetime value calculations (Gupta, Lehmann, and Stuart 2004; Berger and Nasr 1998). The lost-for-good approach has been criticized by Rust, Lemon, and Zeithaml (2004), who proposed an alternative *migration* approach in which the customer leaves for a limited time—possibly to a competitor—and then may return.

An interesting question relates to the kind of attrition that disadoption represents. Hogan, Lemon, and Libai (2003) modeled both a case in which the customer is lost for good once he or she disadopts and a case in which, after leaving the service, the customer may come back during the growth process. The latter case is probably more realistic for most innovative services. Longitudinal improvements, especially in terms of service price and quality, coupled with reduced uncertainty and growing social pressure to adopt, render a customer returning to a service previously dropped a reasonable expectation. Indeed, one reason for considerable investments in online banking in the late 1990s was the realization that at the time, attrition occurred due to the low utility of the service in its initial form. But when it became more user-friendly and functional, those who tried it and were disappointed returned (Monahan 2000).

Category-level Services Growth Model with Attrition

In this section we introduce a category-level model for services that incorporates disadop-

Figure 1

Diffusion Pattern under the Assumption of (A) Customers Lost for Good and (B) Customers Who Stopped Using Service Rejoin

(A)

$m - N(t) - R(t)$ potential	q p	$N(t)$ users		$R(t)$ non-users

δ

(B)

$m - N(t)$ potential	q p	$N(t)$ users

δ

tion and use the model to understand how the disadoption rate affects the growth of a new service category and the consequent managerial implications. Following the discussion in the previous section, there are two options available to model attrition in general and disadoption in particular: The first is lost-for-good disadoption in which customers who disadopt will never join the service again. The second is a model in which a disadopter may rejoin the service later on. Figure 1 illustrates the diffusion pattern under each model: while in lost-for-good disadoption, customers who leave are removed from the market, the alternative assumption regards adoption as a periodic process in which customers who leave join the pool of potential users.

From a dynamic modeling point of view, the lost-for-good option is problematic, since the constant disadoption leads to a zero level of adoption in the long run, regardless of the values of the rest of the parameters. This fact is inconsistent with classical diffusion approaches as well as empirical data. In addition, as mentioned earlier, anecdotal evidence supports the option of a customer who might eventually return. Hence, in our model, and consistent with the calls to take eventual customer return into account when modeling attrition (Rust,

Lemon, and Zeithaml 2004), we assume that disadopting customers can rejoin.

Giving disadopters the possibility to return, one needs to decide how to model the return probability of disadopting customers, compared with the probability of acquiring new ones. There is no empirical evidence that supports a clear assumption on this point. If disadopters wait for the service to improve in order to rejoin, they will need advertising and word-of-mouth communication, as do those who have not yet adopted. In the absence of information on a difference, and to allow a simple and parsimonious approach, we assume that the probability of acquisition of a customer who left is equal to the probability of gaining a customer who has not yet adopted the service.

Theoretically, while a customer might return right after disadoption, Figure 1 implies that the average readoption will take a long time. The reason is that when being a part of the potential users pool, the individual's return is subject to the diffusion process and the probabilities are according to the diffusion parameters; therefore the return is not immediate.

The empirical literature does not yet provide much data on the variation of attrition, either disadoption or churn, over time. Past research in this area has generally used a constant value of attrition over time (see a discussion on this point in Gupta and Lehmann 2003). There is anecdotal evidence, however, that suggests that for some new services, disadoption rates may be higher initially. For example, in the case of cellphones, disadoption rates started with high values, yet decreased to 2%-5% over time. We did examine a more complex version of our model in which disadoption rates decrease exponentially with time. Using both formal and numerical tests, we found that the basic equations can be solved analytically, and that the qualitative theoretical results presented in the following sections regarding the effect of disadoption on the market potential and the time to maximum growth hold for the case

of disadoption rates decreasing exponentially with time. Hence, consistent with previous approaches, we also assume a constant value of attrition over time.

Note that we have made some simplifying assumptions in order to keep the model parsimonious; some can be relaxed without a fundamental change in the basic logic of the model. In the last section of the paper, we discuss the possible implications of relaxing or changing the assumptions. We define the following variables and parameters:

δ Disadoption rate from the category

p External influence parameter

q Internal influence parameter

$N(t)$ Number of subscribers at time t

m Market potential

The diffusion of the new service is thus given by the following equation:

$$\frac{dN(t)}{dt} = p(m - N(t)) + \frac{qN(t)}{m}(m - N(t)) - \delta N(t) \quad (1)$$

Equation 1 is a first-order quadratic differential equation. Using the initial condition $N(0) = 0$, this equation can be integrated to arrive at the following solution (the derivations are given in the Appendix):

$$N(t) = \frac{\bar{m}(1 - e^{-(\bar{p}+\bar{q})t})}{1 + (\bar{q}/\bar{p})e^{-(\bar{p}+\bar{q})t}} \quad (2)$$

$$\bar{m} \equiv m\frac{\Delta + \beta}{2q}, \bar{p} \equiv \frac{\Delta - \beta}{2}, \bar{q} \equiv \frac{\Delta + \beta}{2},$$

$$\beta \equiv q - p - \delta, \text{ and } \Delta \equiv \sqrt{\beta^2 + 4qp}.$$

Interestingly, the penetration curve in Equation 2 has the same functional form as the Bass equation (1969), but with different parameters: $\bar{q}, \bar{p}$, and $\bar{m}$, instead of q, p, and m. Unlike p and q of the Bass model, $\bar{p}$ and $\bar{q}$ are not independent and do not represent fundamental probabilities. We present this formula-

tion only in order to illustrate the similarity to the Bass diffusion function. It can be easily shown that both β and Δ decrease with respect to δ; hence $\bar{m} < m$, $\bar{p} > p$, $\bar{q} < q$, and all three parameters are positive. When the disadoption rate δ is zero, it is easy to see that Equation 2 converges with the Bass diffusion function.

An important implication of Equation 2 is that in the presence of disadoption, the maximum number of subscribers $\bar{m}$ is lower than the market potential m. Since customers are constantly leaving, the service cannot exploit the real market potential m, but rather it approaches an *effective market potential* $\bar{m}$, which is smaller than m and decreases with the increase of the disadoption rate. In order to increase this effective market potential and make it closer to the real one, firms have to invest in reducing disadoption. If the online banking industry, for example, reduces attrition from 16% to 5%, it will gain over 8 million additional subscribers. Thus, while the traditional CRM literature sees the benefits of reduced disadoption in increasing the lifetime value of a single customer through retention, our modeling illustrates the additional gains in terms of acquisition of additional customers.

The existence of an effective market potential raises the issue of the interpretation of market potential: it can be viewed either as the number of those who will ever try the product or as the level of market saturation. When there is no disadoption, both interpretations coincide. However, in the presence of disadoption, these are two different constructs: m is defined as the number of people who potentially will ever try the service. The level of market saturation is $\bar{m}$, the effective market potential, which is always lower because of attrition.

Given this sensitivity of the diffusion process to the disadoption level, one might wonder to what extent ignoring attrition when modeling service growth will bias the parameter values. Assume a growing service category whose dif-

fusion parameters are estimated from continuous service (e.g., subscriber) data and that the service has a disadoption rate δ. As we have shown earlier, the penetration curve described in Equation 2 is equivalent to the Bass curve with $\bar{p}$, $\bar{q}$, and $\bar{m}$ instead of p, q, and m. Therefore, a researcher who estimates the parameters using the Bass function aims to estimate p, q, and m, but actually estimates $\bar{p}$, $\bar{q}$, and $\bar{m}$. This discrepancy leads to a bias in the parameter estimation: using the definitions of $\bar{p}$, $\bar{q}$, and $\bar{m}$, it is straightforward to show that q and m are underestimated, and p is overestimated.

In order to demonstrate the magnitude of the bias, we used data from three service categories evaluated in previous diffusion studies: cellphones (Krishnan, Bass, and Kumar 2000; Lilien, Rangaswamy, and Van den Bulte 2000), cable TV (Lilien, Rangaswamy, and Van den Bulte 2000), and online banking, all in the United States (Hogan, Lemon, and Libai, 2003). For each service category, we obtained the historical data on the number of subscribers, using industry data and financial reports (10K and 10Q). We used nonlinear least-square estimates (Putsis and Srinivasan 2000) to evaluate p, q, and m of Equation 1.

The disadoption rates (δ) are constantly monitored by the firms or by industry analysts, and therefore we use them as given instead of estimating them empirically. We used industry data and previous literature to obtain values of the disadoption level δ for each industry. While average total attrition for U.S. mobile carriers is 25% (see, for example, Hawn 1999), analysts with whom we spoke estimated annual disadoption at 1% to 4%; we used an average value of 2%. In the U.S. cable TV industry, monthly attrition varies between 1% and 4%, with an average of 1.5% (see, for example, the annual reports of the FCC, or Kramer 2002). Since many cable TV operators are local monopolies, the value of 1.5% monthly—compounded annual attrition rate of 16.5%—can be used as an approximation for the disadoption rate. For online banking, we used 16%, since this is considered the disadoption rate among involved, bill-paying customers (see, for example, O'Sullivan 2000).
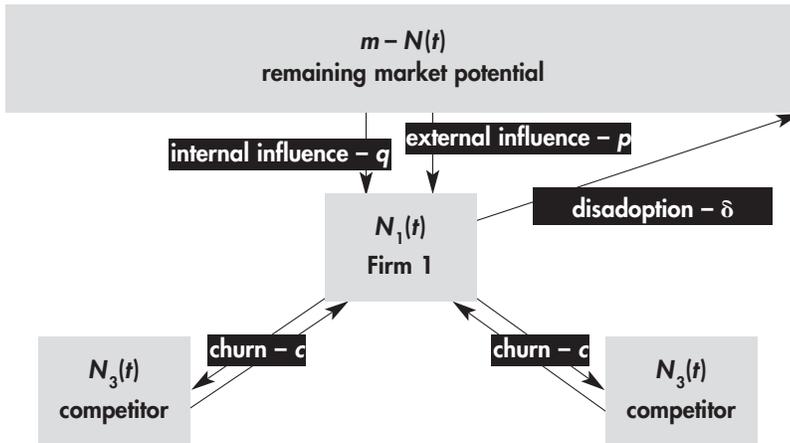
Table 1 displays the results of the analysis, which imply that the bias is considerable: for the services in Table 1, the average overestimation of p is 41%, and the average underestimation of q and m is 25%. The major managerial effect of this bias is in diagnostics. We demonstrated that neglecting attrition may lead to misinterpretation of the effective market potential $\bar{m}$ as the real market potential, where in effect this number can increase with the industry's investments in reducing disadop-

Table 1

Bias in the Diffusion Parameters for Three Service Categories

Category	Years	Bass Model			Services Growth Model				Bias			R^2
		p	q	m millions	Disadoption δ	p	q	m millions	p	q	m	
Cellular phones–U.S.	1984–2004	.0030 (.0024)	.364 (.012)	209.1 (4.48)	2%	.0029 (.0023)	.384 (.013)	220.5 (4.71)	5%	–5%	–5%	.935
Cable TV–U.S.	1961–2004	.0029 (.0033)	.174 (.0039)	74.7 (1.49)	16.5%	.0015 (.0017)	.328 (.0073)	140.6 (2.82)	88%	–47%	–47%	.497
Online banking–U.S.	1994–2003	.0142 (.0047)	.545 (.03)	42.9 (9.99)	16%	.011 (.0061)	.702 (.025)	55.3 (7.79)	29%	–22%	–22%	.812

Standard errors are in parentheses.

defect to competitors. If the attrition rate of firm i is denoted by a_i, then it consists of disadoption and churn, in an additive form: *total attrition rate* (a) = *disadoption rate* (δ) + *churn rate* (c). Figure 2 illustrates the customer flow to and from a focal firm in a three-firm market.

Let $N_i(t)$ be the number of subscribers of firm i at time t. The total number of subscribers in the category is $N(t) = N_1(t) + N_2(t) + \ldots + N_k(t)$. Let p_i be a parameter representing the power of external influence (advertising and other marketing efforts), while q_i represents the power of internal influence (typically word-of-mouth and imitation). Similar to conventional diffusion modeling, we assume that word-of-mouth is exchanged between users and nonusers.

tion. Another misinterpretation concerns the values of the estimated p and q. Since p is usually regarded as influenced by the advertising policy of the firm, the biased p may lead firms to overestimate the influence of their current advertising and, as a consequence, to underinvest in advertising. Similarly, the biased q may lead to undervaluing the power of the word-of-mouth influences in the industry. Finally, neglecting attrition may be problematic when comparing penetration curves of countries, or industries, which differ in their disadoption rates. In such cases, differences of curves may be related to the p, q, and m, where at least some of the differences are due to the different disadoption rates.

We also assume that internal influences are at the brand-specific level, that is, potential users adopt a brand due to communication with the existing customers of the brand alone. In this sense, we take the approach of Mahajan, Sharma, and Buzzell (1993), and Kalish, Mahajan, and Muller (1995). Under the above assumptions, the diffusion, which is graphically illustrated in Figure 1, can be described using the following competitive services growth model for firm i:

$$\frac{dN_i(t)}{dt} = p_i\big(m - N(t)\big) +$$
$$\frac{q_i N_i(t)}{m}\big(m - N(t)\big) - a_i N_i(t) +$$
$$\sum_{j \neq i} c_j N_j(t)/(k-1) \tag{3}$$

where $a_i = c_i + \delta_i$. The division of the last term by $(k-1)$ is needed, as we assume that customers who defect from one competitor are equally divided among $k-1$ competitors. Note, that we could alternatively assume that the distribution of churning customers is done according to $N_i(t)$—the number of subscribers for each firm—to the growth rate $dN_i(t)/dt$, or to the advertising investments of the firms,

Competitive Services Growth Model

We next present a model that describes the growth of a service firm, taking into account the two forms of attrition: churn and disadoption. Consider a firm that introduces a new service into a market with potential m, with k competing firms in the market. At every time t, there are customers who stop using the new service: while some of them disadopt, others

represented by p_i. While the best method of new customer partition depends on the specific market, lacking any empirical generalization on that issue and for the sake of modeling simplicity, we decided on equal distribution.

One might inquire about the relationship between this model and the previous one, described in Equation 1, that dealt with the category level. If one assumes that the attrition levels are the same for all firms (an assumption supported at least by our data), then summing up Equation 3 for all firms, and rearranging terms yield the following equation:

$$
\frac{dN(t)}{dt} = p\big(m - N(t)\big) + \\
\sum_i q_i N_i(t) \frac{\big(m - N(t)\big)}{m} - \delta N(t) \qquad (1')
$$

where $N(t) = \sum_i N_i(t)$, and $p = \sum_i p_i$.

Suppose we were to rewrite Equation 1 but with p and q that are related to the individual firm level by the following: $p = \sum_i p_i$ and $q = \sum_i q_i/k$, i.e., the internal parameter q of Equation 1 is the average of the individual firms' internal parameters. One might ask the question about the difference between the two equations (1 and 1′) from a practical point of view. The answer is that the difference is surprisingly small. In comprehensive simulations that we conducted on the numerical solutions of the equations, we found that relatively large differences in q_i will lead to small deviations in dN/dt as calculated from equations 1 and 1′. For example, if we take the average values of the parameters in our data set (quarterly data and annual averaged separately) we see that for the annual data, while the ratio of the largest to the smallest q is 2, the percent average deviation of dN/dt as calculated from equations 1 and 1′ is 4.2. For the quarterly data, the ratio is 2.8, while the percent difference is 12.6%.

The equation system presented in Equation 3 can be solved analytically, under some restrictive conditions (the solution is available from

the authors). The solution is an S-shaped function, which is similar to the penetration function of Equation 2, with an additional term that describes the balance between a firm's effectiveness in attracting adopters, and the attrition's components.

The Customer Equity of Competitive Firms

In this section, we present an application of increasing interest among researchers and practitioners: calculating the customer equity of firms. In CRM terminology, customer equity is the sum of the customer lifetime value (CLV) of the firm's customers (Rust, Zeithaml, and Lemon 2000). Customer equity is increasingly regarded by both practitioners and academics as a key measure for the success of a firm's operations with its customers (Peppers and Rogers 2005; Rust, Lemon, and Zeithaml 2004). It can be used, for example, in order to determine the effectiveness of marketing mix and service activities, optimizing the tradeoff between investments in customer acquisition and retention, or when examining the effect of operational measures such as satisfaction or attrition on the firm's long-term profitability.

While initial approaches for the calculations of customer equity focused on profit stemmed from existing customers (Blattberg, Getz, and Thomas 2001), later work defined customer equity as the discounted sum of profits from present *and future* customers (Rust, Lemon, and Zeithaml 2004). Indeed, for growing firms, the contribution of future customers to equity can be a significant part of the firm's overall equity and thus requires estimating the expected growth in the number of customers.

In the first attempt to rigorously examine the customer equity of a growing service firm, Gupta, Lehmann, and Stuart (2004) suggested a method for calculating based on publicly available data such as the number of sub-

scribers, margins, and retention rates (see also Gupta and Lehmann 2005). Consistent with the CRM literature, Gupta, Lehmann, and Stuart calculated the CLV of a single average customer of the firm and then summed up the CLV of existing and future customers. For firm i, let the acquisition costs of a single customer be denoted by $cost_i$, the lifetime value of a single existing customer be CLV_i, $N_i(t)$ the number of customers of firm i at time t, and $n_i(t)$ the number of customers who joined during period t. Finally, let the discount rate be denoted by ρ. Customer equity of firm i at time t is given by the following:

$$Customer\ Equity_i(t) = N_i(t) \cdot CLV_i +$$
$$(CLV_i - cost_i) \int_{s=t}^{\infty} n_i(s) \cdot e^{-\rho(s-t)} ds \qquad (4)$$

The first term on the right-hand side of Equation 4 is the contribution of the existing customer base, and the second term is the summation over all future customer cohorts, discounted according to the time difference between the starting point t and the beginning of the revenue stream from the customer. Hence, customer attrition may have a *dual effect* on customer equity: First, it influences the individual customer's CLV. Second, following our discussion in the previous section, it affects the shape of the diffusion curve, as expressed by $n_i(t)$. The value of Equation 4 depends on the functional shape of the growth curve. Gupta, Lehmann, and Stuart applied their model to data containing the number of individuals who ever adopted the product and not number of current subscribers; therefore they did not relate explicitly to attrition or to competitive effects. We are interested in capturing the influence of both attrition and competition; therefore we apply Equation 4 where $n_i(t)$ is derived from the competitive services growth model (Equation 3). One should note that with some restrictive conditions Equation 4 can be formally calculated to yield a solution involving the Gauss $_2F_1$ hypergeometric function (available from the authors upon request).

The competitive approach presented above is consistent with recent research that cautioned against the widespread use of lost-for-good retention measures when calculating customer equity (Rust, Lemon, and Zeithaml 2004). Rust, Lemon, and Zeithaml argue that since customers may come back, a lost-for-good approach understates customer equity. Thus, they used a Markov switching matrix to analyze customer equity. This approach demands the use of detailed market research to capture individual-level data on switching probabilities among different brands and better fits mature service industries where the diffusion dynamics are not taken into account. Our approach is in the same spirit, but in the context of new service growth, so the switching process is embedded in the diffusion model. It is also based on aggregate data that are typically more accessible. Note, however, that Figure 2 and Equation 3 can be regarded as describing a reduced case of a Markov model, from the point of view of a single focal firm, with a single-step transition and symmetric distribution of churning customers.

In the following section, we demonstrate the dual effect of attrition on customer equity: We calculate the customer equity of five service firms using the growth function of the services growth model and compare the results to the stock market value of these firms (see Gupta, Lehmann, and Stuart 2004). This comparison is of interest to both finance and marketing researchers, since it illustrates the importance of a model which explicitly considers attrition of customers, and shows to what extent, and in which cases, the stock market valuation agrees with or deviates from the straightforward customer equity approach presented here.

Empirical estimation of customer equity in a competitive environment

Calculating the equity in a competitive scenario requires a comprehensive set of data on the market evolution. Since it involves the estimation of the diffusion parameters of both the focal firm and the category, historical sub-

scriber data should be obtained at the firm- and category-levels. This requires, in many cases, collecting data from all of the players in the category.

More complex is the comparison with the stock market value: First, the firm has to be public. Second, the firm should operate and be traded in a single competitive market. For example, the competitors of the global mobile operator Vodafone are local for each country, i.e., Vodafone Netherlands competes with the Dutch operators, while Vodafone Spain competes with the Spanish operators. However, the Vodafone group is traded globally on the NASDAQ; therefore the connection between the global valuation and the growth of one of its local branches is unclear. Third, the new service has to be a significant part of the firm's activity. The online brokerage firm Charles Schwab, for example, competes in the U.S. online brokerage market. However, it has many additional activities. Similarly Bouygues Telecom is the third largest mobile operator in France, yet its mobile business accounts for only 16% of its total activity.

Hence, there are several limitations on the type of firms that can be used for our study. A recent example of an industry in which attrition plays a dominant role is cellular service operators. To study the cellular market in this respect, we used data from the World Cellular Information Service (WCIS), a major data provider for this industry. Aiming to study the European cellular market, which includes 16 countries and over 50 operators, we found that one operator—the Belgian operator Mobistar—matches our requirements. A similar procedure was conducted in Asia Pacific, which resulted in one Korean operator—SK Telecom (South Korea Telecom).

Overall, we use data on 11 firms: 5 focal firms in 4 markets and their main competitors—Amazon.com, Barnes&Noble.com, E*Trade, Mobistar, and SK Telecom. The competitors of Mobistar (traded on the Brussels Stock Exchange) are Belgacom and BASE. The competitors of SK Telecom (NYSE) are the KT group and LG Telecom. In the fragmented United States, online brokerage industry reports and E*Trade's (NYSE) own analysis suggest that Ameritrade and Schwab are E*Trade's main competitors. In principle, Ameritrade could also be used as a focal firm; however its growth is achieved to a large extent through mergers and acquisitions, and its penetration curve, especially in the last two years, does not represent organic growth. Amazon.com (NASDAQ) was studied with its major competitor, the online services of Barnes & Noble (Barnes&Noble.com was traded on the NASDAQ until it was purchased by Barnes & Noble in the third quarter of 2003). Although Amazon.com has business lines other than books, book retailing forms most of its revenues and Barnes&Noble.com is considered to be Amazon's main competitor in this market (Filson 2004; Mutter 2003). Amazon.com and E*trade were also studied in Gupta, Lehmann, and Stuart (2004), but we enhanced the data so as to include competitors' data and extended the data until the first quarter of 2005.

We obtained customer and financial data from financial reports, 10K and 10Q forms, press releases, and the WCIS data provider. Following Gupta, Lehmann, and Stuart (2004), the margins and the acquisition costs were taken as the average over the last four quarters. The attrition rates were taken from firms' reports and from Gupta, Lehmann, and Stuart (2004). Table 2 summarizes the data for each firm.

For each industry and for firm i, the diffusion parameters p_i, q_i, m, and c were estimated using Equation 3, and Equation 1 for the industry as a whole, using seemingly unrelated nonlinear least squares (SAS "proc model," with SUR option). The estimation was performed simultaneously for all the firms within each industry. Recall that the overall attrition rates are constantly monitored by the firms,

Table 2
Descriptive Data for the Five Focal Firms

Focal Firm	Competitor(s)	Data Period		Customers (millions)	Quarterly Margin per Customer ($)	Acquisition Cost per Customer ($)	Annual Attrition
		From	To				
Amazon.com (USA)	Barnes&Noble.com	Dec. 1996	Mar. 2005	44	6.0	8.3	30%
Barnes&Noble.com (USA)	Amazon.com	Sep. 1997	Dec. 2004	21	1.1	4.5	30%
E*Trade (USA)	Ameritrade Charles Schwab	Dec. 1997	Mar. 2005	3.6	51.8	248.3	5%
Mobistar (Belgium)	Belgacom BASE	Jan. 1996	Mar. 2005	2.8	109.1	181.8	23%
SK Telecom (South Korea)	KT group LG Telecom	Jan. 1984	Mar. 2005	18.8	79.1	169.2	27%

Number of customers, quarterly margins, and acquisition costs are calculated for December 2004, except for Barnes&Noble.com for which quarterly margins and acquisition costs are the latest available (September 2003—the time of acquisition).

and therefore we used them as given. Although the model in Equation 3 allows for different attrition and churn rates among firms, in our analysis the attrition and churn rates (and therefore the disadoption rates) were taken as identical among the competitors and equal to that of the focal firm. The reason is that while our attrition data are available for the focal firms, they are incomplete for the rest of the firms. Moreover, from trade publications and from the limited data we do have, we see that the attrition rates of firms within the same industry are quite similar. For the two cellular cases, where we had more attrition data for the nonfocal firms, we also ran the analysis by allowing the attrition and churn rates to vary among the competitors. The results in terms of the other parameters were similar. However, almost all of the disadoption parameters were found to be nonsignificant.

Table 3 presents the parameter estimations for the eleven firms based on Equation 3.

Note that for the cellular operators, the differences between churn rates and overall attrition

implies high disadoption rates relative to what we usually expect in the cellular industry. The reason is that in the early days of the cellular industry, disadoption rates were much higher then the current values of 2% to 5%. Since we use a constant disadoption rate, the estimation we get is an average over time.

Having estimated the parameters, the customer equity was calculated for the five focal firms using Equation 4 (see the explanation above for why the available data do not enable the equity calculation for all 11 firms). When calculating the CLV, we assumed a long-term planning horizon, that is,

$$CLV = \sum_{t=1}^{\infty} \frac{g \cdot r^t}{(1 + \rho)^t} = \frac{g \cdot r}{(1 + \rho - r)} = \frac{g \cdot (1 - a)}{(\rho + a)}$$

(g is the gross profit margin; ρ is the discount rate, r is the retention rate; $a = 1 - r$, see Berger and Nasr 1998). As in Gupta, Lehmann, and Stuart (2004), we used a 12% discount rate, deducted the relevant corporate tax rate (38%

Table 3
Parameter Estimations Based on Equation 3, First Quarter 2005

Category	Firm	p_i	q_i	Churn Rate c	m	R Square
Online Books[+]	Amazon	.00489*	.168**	2.3%**	138**	22.4%
	B&N.com	.00075	.138*			42.0%
Online Brokerage[+]	E*Trade	.0019	.373**	1.3%	11.2**	53.0%
	Schwab	.0206*	.0888*			53.3%
	Ameritade	.00204	.316*			88.1%
Cellular Belgium	Belgacom	.0	.988**	8.0%**	9.9**	89.7%
	Mobistar	.00503	.989**			83.2%
	Base	.0053	.889*			48.6%
Cellular Korea	SK Telecom	.0027	.754**	13.0%**	50.2**	63.4%
	KT Group	.017*	.569**			67.3%
	LG Telecom	.0086*	.0			57.0%

[+] The value of p, q and churn refer to quarterly data. * Denoted significant at 5%, while ** significant at 1%.

for the U.S. firms, 30% for Mobistar and SK Telecom) from the equity, and used the after-tax value as a proxy for the firm value.

In order to study the effects of attrition on customer equity, we compared our calculations to a competitive model that does not consider attrition. Hence, we reestimated the parameters using Equation 3, taking $a = c = 0$, and calculated the equity. This "no attrition" model is close in spirit to that of Kalish, Mahajan, and Muller (1995), and to the model of Krishnan, Bass and Kumar (2000) with brand-level word-of-mouth instead of category-level word-of-mouth.

Table 4 presents the calculated customer equity for the five focal firms based on Equation 4, with the penetration function of Equation 3, compared with the calculations using the model that does not consider attrition, for the first quarter of 2005. For each focal firm, it presents the estimated market potential and the calculated value for March 2005.

The results imply that for all firms, the valuation of customer equity according to the competitive services growth model is considerably higher than that of a model without attrition, especially for the high attrition rates. The main reason for this is that when attrition is non-zero, then $n_i(t) = dN_i/dt + aN_i(t)$, whereas a zero attrition model uses $n_i(t) = dN_i/dt$. That is, we consider the contribution of *all* those customers who joined the service during the period. When considering only $n_i(t) = dN_i/dt$ (namely the net difference in number of subscribers between periods), their contribution is ignored.

Adjusting the data and adding the customers who left could at least partially compensate for the use of a no-attrition model. For a monopoly, such adjusted data are the number of individuals who ever adopted the service. However, this adjustment provides only a partial compensation, since it does not contain the accumulated word-of-mouth contribution of these customers. In a competitive scenario, such adjustment is problematic, since concurrent to adding the customers who left, one should subtract the customers arriving from competitors. Such subtraction requires a prior knowledge of churn and disadoption, and in addition, the interpretation of the adjusted data becomes unclear.

Table 4
Customer Equity

Firm	Competitive Services Growth Model		Competitive Model Without Attrition	
	Market potential (millions of subs)	Value ($ millions)	Market potential (millions of subs)	Value ($ millions)
Amazon.com	138	5,308.9	86.1	1,611.3
Barnes&Noble.com	138	342.3	86.1	131.6
E*Trade	11.2	3,675.9	11.1	2,815.5
Mobistar	9.9	4,813.7	8.3	1,688.5
SK Telecom	50.2	18,214.2	36.5	7,010.1

All data are valid for March 2005, except for Barnes&Noble.com which has data for September 2003 (date of merger).

Figure 3 presents a comparison between our valuation, the valuation of a model without attrition, and the average stock market value of the firms. The comparison was performed for the end of the third and fourth quarters of 2004 and the first quarter of 2005. Since Mobistar provides full operational reports only once a year, we performed comparisons for Mobistar for the fourth quarters of 2002, 2003, and 2004. Barnes&Noble.com was an independent public company that was traded on the NASDAQ until 2003. In the third quarter of 2003, it was purchased by Barnes & Noble. We performed the equity calculation of Barnes&Noble.com for the time of the acquisition.

Figure 3 has a number of implications. First, in all categories, using the competitive services growth model provides estimations that are considerably closer to the stock market value as compared with the model that does not consider attrition. In four of the five firms (the exception is Amazon.com), the customer equity estimations are remarkably close to the stock market value. If we take the latest valuations for Barnes&Noble.com, E*Trade, Mobistar, and SK Telecom, we find the deviation of our calculated valuation from that of the stock market is 17.1% on average.

The one exception is Amazon.com, which is traded at notably higher values than are our valuations. It may be that Amazon.com is val-

ued by the stock market following expectations for growth through means such as synergic mergers and acquisitions or entry of new product markets that are not captured in our model.

Our approach allows us to shed some light on this issue. Barnes&Noble.com, Amazon's main competitor, was an independent public company that was traded on the NASDAQ from its inception in the third quarter of 1997 until 2003. In the third quarter of 2003, it was purchased by Barnes & Noble for $410 million. We performed the equity calculation of Barnes&Noble.com for the time of the acquisition, with the resultant equity of $342 million. At the same time (Q3 2003), Amazon.com was traded for $21,348 million while our calculations—the same ones that were quite accurate with respect to Barnes&Noble.com—came to only $4,469 million. The question that we raise, but do not solve is, why the difference? In the case of Barnes&Noble.com, we came up short by less than 17%, while with Amazon.com, the exact same calculations that used the same publicly available data underestimated the stock market by 79%. While the reasons for this discrepancy are outside the scope of this paper, this analysis highlights the fact that this difference might not necessarily stem from the gap in the customer equity of customers frequenting these two retailers.

Figure 3
Market Value and Customer-based Valuation
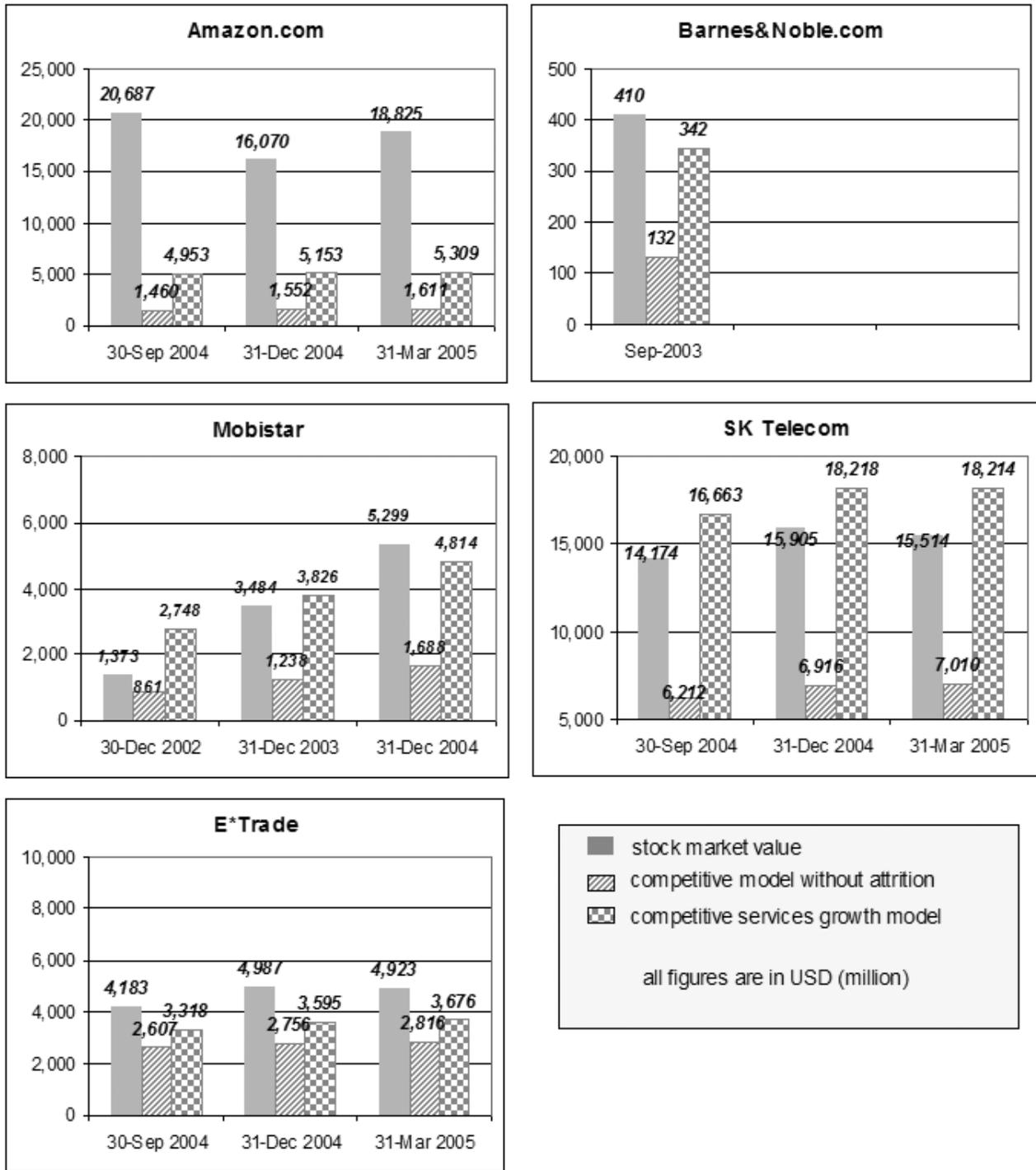
Discussion

At the beginning of this paper, the growth of competitive services was compared to a leaking bucket: There is an inward flow of customers, either new adopters or customers who switch from the competitors. There is also an outward stream of customers who either disadopt the

category or defect to the competition. This complex environment makes the analysis of the growth of new services nontrivial; yet the ubiquity and the importance of new services makes this task essential for managers wishing to understand the environment in which their services grow.

We presented a competitive services growth model that can serve as a platform for this analysis. Our approach is relatively straightforward, and with a few simplifying assumptions, the basic model has a closed form solution. To demonstrate its possible application, we focused first on the category level and examined the role that category-level attrition plays in the evolution of markets for new services. We demonstrated how using a durable goods approach to study the growth of services can considerably bias the estimation of the parameters of growth.

We then moved to the firm level and used our approach to develop a functional solution to the customer equity of firms. The approach presented here is the first customer equity measure that takes into account interfirm dynamics in a growing market and is especially critical to the calculation of customer equity where firms are highly affected by both customer switching to the competitors and by disadoption of the category.

As firms aim to better understand the economic value they give to their shareholders, there is increasing interest in the marketing/finance interface in general (e.g., Kumar and Petersen 2005; Hogan et al. 2002) and specifically in the relationship between market-based assets and the value created to shareholders (Srivastava, Shervani, and Fahey 1998). Hence there have been recent efforts to contrast customer equity and stock market value of service firms (Gupta, Lehmann, and Stuart 2004). A match-up between the two is not always straightforward, especially for firms where long-range customer value is just one of the sources of shareholders' value. Yet, for service firms whose value is derived mostly from customers, the comparison is relevant, and can help researchers to better understand the role customer equity plays in the perceptual value of firms.

We used our approach on five firms in three markets. As Figure 3 demonstrates, we found that with the exception of Amazon.com, customer equity as we estimated it was remarkably close to the stock market value of four firms, with an average difference of about 17%. Interestingly, while our approach was quite close to the stock market approach, a competitive model with no attrition taken into account yielded much lower valuations. With the necessary caution stemming from our small sample, we posit that customer equity may play a critical part in the way the stock market values service firms. This may turn out to be an important aid to those advocating the proper management of customer assets as a way to increase shareholder value and, in a more general sense, as evidence of the role of marketing in the firm.

The role of customer equity measurement is also apparent when analyzing the case of the one firm that did not match our valuation well: Amazon.com. The value of Amazon.com stock has been the subject of much industry-related discussion since the late 1990s, hence the need to examine the relationship between the basis of our assumptions and the market value of the firm. A good question is to what extent a firm's market value is based on the current performance of the firm, given the growth pattern we have seen to date. This is what the customer equity measure does, and we see that, at least based on the data that we used, the stock market valuation reflects something over and above customer equity. One possibility is expectations regarding income from sources other than customers, such as web-based advertising. Another option is that the stock market expects a change in one of the basic customer equity parameters, for example, a change in the average profit per customer per period due to successful cross-selling.

The customer equity framework can be used as a tool to investigate the market's expectations. For example, in the assumption relating to rising per-customer profit, one can ask what should be the average profit per customer of Amazon.com in the coming years that will push the customer equity that we estimated close to the stock market value. Based on the data we used, the average profit should be about $20 per quarter as opposed to the current $6. In the same fashion, the market potential reflected in the stock market valuation of Amazon.com should be about half a billion customers versus the current market potential of 130 million.

Our data enable us to shed light on another interesting issue. Together with Amazon.com, we analyzed the customer equity of its largest competitor, Barnes&Noble.com. While Barnes&Noble.com's value was 17% off from our customer equity measure, Amazon.com's was 79% off. Many of the industry-related growth assumptions relevant to Amazon.com should be relevant to Barnes&Noble.com as well, so the difference might not be a function of only the customers' valuation. Thus the question remains: What are the assumptions underlying the difference?

Limitations and conclusion

The services growth model relies on a number of assumptions, mainly related to the nature of attrition. The assumptions were made in order to provide an analytical formulation, yet numeric simulations that we conducted can provide direction on the consequences of relaxing some of these assumptions. Following the empirical evidence, we assumed that customers who disadopt innovative service technologies eventually rejoin the service. As discussed previously, this seems a much more realistic assumption than a lost-for-good one. However, in real markets, some customers leave the service permanently and are truly lost for good. Using the customer pool analogy of Figure 1, such a scenario is described by an additional outward customer flow that permanently leaves the system, instead of

reentering the pool of potential users. Our simulations indicate that such a flow will cause a decline in the effective market potential, which in turn will hinder diffusion and lower customer equity.

Another assumption is the equal probability of return of disadopting customers compared with the acquisition probability of new ones. This assumption means that although customers differ in their p and q, we consider the average values of their probabilities. Using multiple probabilities requires additional parameters p_1 and q_1 to describe the return probabilities of disadopters. In such a case, the model is expanded to an equation set that cannot be solved analytically. Numerical simulations indicate that using group-specific values instead of average values does not change resulting market behavior.

Our approach does not take negative word-of-mouth into account and in that sense is consistent with most of the diffusion literature. It might be, for example, that the parameter q used in the diffusion literature (and here) to describe internal effects due to previous adopters already represents the net positive internal effects, after taking both positive and negative effects into account. Other approaches may model negative word-of-mouth through the size of the market potential. In this paper we limit ourselves to modeling customer attrition and do not cover the wider topic of the consequences of customer dissatisfaction. While both are often related, they are not identical. Dissatisfaction can cause other effects besides attrition such as negative word-of-mouth or share-of-wallet change. Attrition may be affected by dissatisfaction, but also by other factors such as switching costs or a change in customer needs.

This study opens wide options for theoretical and empirical research to further enhance this direction. The dependence of the attrition rate on penetration time can be measured and

explicitly incorporated into the model. Moreover, empirical investigation as to the word-of-mouth distribution of disadopters can validate (or modify) the model's basic assumptions.

Since the 1990s, one can see in the marketing literature a clear direction toward studying customer attrition in order to understand its implications for marketing strategy. The incorporation of customer attrition into mainstream marketing models is part of the shift of the marketing discipline from the study of marketplace exchanges as transactions to that of relationships that need to be managed and examined for the long term (Agustin and Singh 2005; Morgan and Hunt 1994). For that, marketers should adapt the tools they use, and we hope that this study can serve as a step in this direction.

Acknowledgments

Appendix: Solving the Aggregate Diffusion Equation

Summing up the equations of the competitive services growth model for the individual firms, the category growth is described by the following equation:

$$\frac{dN(t)}{dt} = p(m - N(t)) + \frac{qN(t)}{m}(m - N(t)) - \delta N(t) \quad \text{(A.1)}$$

Equation A.1 is equivalent to:

$$\frac{dN(t)}{dt} = \frac{-qN^2(t)}{m} \ (q - p - \delta)N(t) + p \cdot m \quad \text{(A.2)}$$

The right-hand side of the equation is a quadratic polynomial. Its roots are denoted by r_1 and r_2, and are given by

$$r_{1,2} = \frac{-\beta \pm \Delta}{\frac{-2q}{m}} = \frac{\beta \mp \Delta}{\frac{2q}{m}},$$

where $\beta \equiv q - p - \delta$, and $\Delta \equiv \sqrt{\beta^2 + 4qp}$.

Recall that for a quadratic equation with roots r_1 and r_2, $Ax^2 + Bx + C = A(x - r_1)(x - r_2)$.

Thus, we can perform a separation of variables and transform the equation to:

$$\frac{dN}{\left(\frac{qN}{m} - r_1\frac{q}{m}\right)(r_2 - N)} = dt$$

$$\Rightarrow \frac{dN}{(r_2 - r_1)\left(\frac{qN}{m} - r_1\frac{q}{m}\right)} + \frac{dN}{\frac{q}{m}(r_2 - r_1)(r_2 - N)}$$

$$= dt \quad \text{(A.3)}$$

Integrating Equation A.3 under the initial condition $N(0) = 0$, we get:

$$N(t) = \frac{m\frac{\Delta + \beta}{2q}(1 - e^{-\Delta \cdot t})}{1 + \frac{\Delta + \beta}{\Delta - \beta}e^{-\Delta \cdot t}} \quad \begin{cases} \beta \equiv q - p - \delta \\ \Delta \equiv \sqrt{\beta^2 + 4qp} \end{cases} \quad \text{(A.4)}$$

Defining $\overline{m} = m\frac{\Delta + \beta}{2q}$, $\overline{p} = \frac{\Delta - \beta}{2}$, and $\overline{q} = \frac{\Delta + \beta}{2}$, Equation A.4 can be written as:

$$N(t) = \frac{\overline{m}(1 - e^{-(\overline{p}+\overline{q})t})}{1 + \frac{\overline{q}}{\overline{p}}e^{-(\overline{p}+\overline{q})t}} \quad \text{(A.5)}$$

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